

# SUMMER WORK PHYSICS

## Head of Department

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## Exam Board

AQA

## Specification

7407/7408

## COURSE DETAILS

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#### Examination

The course is examined at the end of Year 12 and as a whole at end of Year 13.

Topic 1: Measurements and errors

Topic 2: Particles and radiation

Topic 3: Waves and optics

Topic 4: Mechanics and materials

Topic 5: Electricity

Topics 1 – 5 taught in Year 12. Also in Year 12 students complete 6 required practicals. These will be assessed in public examinations.

Topic 6: Further Mechanics

Topic 7: Fields

Topic 8: Nuclear Physics

Topic 9: Option Topic

Topics 6 – 9 taught in Year 13. Also in Year 13 students complete 6 more required practicals. These will be assessed in public examinations.

## SUMMER WORK FOR INTRODUCTION TO YEAR 12

TASK	TOPIC	
1.	Accuracy of measurements, Magnitudes and Components of Vectors.	<p><b>Complete 20-30 of the questions on each page of the task sheets and self assess these</b></p> <p>Answers are provided so you can check your work as you go (do not simply copy the answers across!)</p> <p>Ideally these should be done in the last week of August so that you are back up to speed with your maths skills.</p>
2.	History of Physics and the Development of Physics	<p><b>Make an A3 poster on the Physicist you feel has influenced the world of Physics the most</b></p> <p>Explain what their contributions to their field did to further knowledge of the scientific community and give your rationale to explain why they are the most influential Physicist.</p> <p>This could be by hand or word processed and emailed for Mrs Lockett to print.</p>
3.	Particle Physics	<p><b>Research the fundamental forces in Physics</b></p> <p>There are 4 fundamental forces, what are their names? Where do they act? What is their carrier particle? What are the ranges of these forces?</p> <p>Write a paragraph on each of the fundamental forces. Do not copy and paste. Give references for where you found your information.</p>

## WIDER READING TO PREPARE FOR COURSE

Suggested Authors and Titles:

Stephen Hawking: A Brief History of Time; The Grand Design

Brian Greene: An Elegant Universe; The Fabric of the Cosmos; The Hidden Reality

Brian Cox: Wonders of the Solar System; Wonders of the Universe; Why does  $E=mc^2$ ?; The Quantum universe

Richard Feynman: QED – The Strange Theory of Light and Matter; Surely You're Joking Mr Feynman

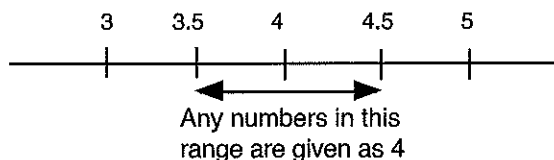
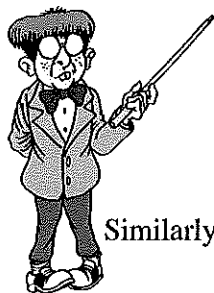


## Accuracy of Measurement Notes.

We have dealt with questions, such as "Round 4.345 to 2 decimal places" and "Round 45683 to 2 significant figures". In this section we are considering these questions in reverse !

When a measurement is given as 4 m, this implies it is measured to the nearest metre. When a measurement is given as 4.0 m, this implies it is measured to the nearest 0.1 of a metre, and when a measurement is given as 4.00 m, this implies it is measured to the nearest 0.01 of a metre. All **these measurements are not precise**. Let us consider the 4 m measurement. If this is to the nearest metre it is possible that the measurement could have been from 3.5 m (**the lower bound**) all the way to 4.5 m (**the upper bound**).

The lower bound fits into our understanding well, if you gave 3.5 to 1 significant figure it would be 4. Notice the upper bound. In our example the upper bound 4.5 to 1 significant figure would be 5. To be more precise the upper bound should really be  $< 4.5$  i.e. 4.49 or 4.499 or 4.4999 etc. This when rounded to 1 significant figure would give 4. These numbers are getting closer and closer to 4.5 so we say that 4.5 is the upper bound.



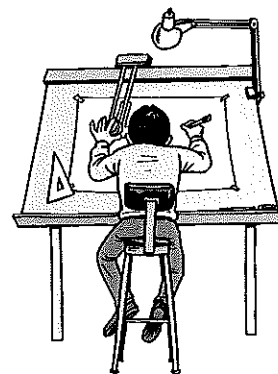
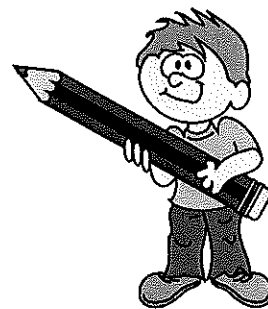
Similarly, 15 to the nearest whole number can be represented by any number from 14.5 to 15.5.

For any value given to the nearest unit

- i). the maximum possible error is half of that "unit".
- ii). its limits of accuracy are the value  $\pm$  half of the "unit".

For example

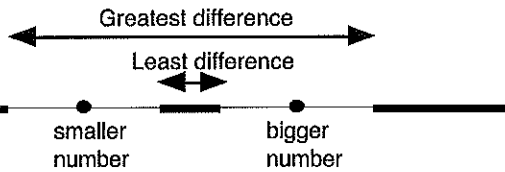
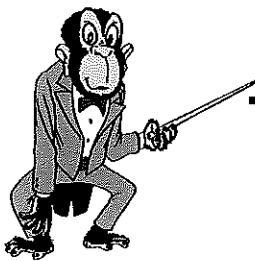
- a). An answer is given as 42 to 2 significant figures.  
So the maximum error is 0.5 (half of 1).  
Its limits of accuracy are  $42 \pm 0.5$  .  
Or lower bound 41.5, upper bound 42.5.
- b). An answer is given as 480 to 2 significant figures.  
So the maximum error is 5 (half of 10).  
Its limits of accuracy are  $480 \pm 5$  .  
Or lower bound 475, upper bound 485.
- c). An answer is given as 73000 to 2 significant figures.  
So the maximum error is 500 (half of 1000).  
Its limits of accuracy are  $73000 \pm 500$  .  
Or lower bound 72500, upper bound 73500.
- d). An answer is given as 8.7 to 1 decimal place.  
So the maximum error is 0.05 (half of 0.1).  
Its limits of accuracy are  $8.7 \pm 0.05$  .  
Or lower bound 8.65, upper bound 8.75.
- e). An answer is given as 6.99 to 2 decimal places.  
So the maximum error is 0.005 (half of 0.01).  
Its limits of accuracy are  $6.99 \pm 0.005$  .  
Or lower bound 6.985, upper bound 6.995.



A value used in a calculation may not be an exact number, it may be an approximation whose level of accuracy and error is known. The approximation may be the result of rounding a number or measuring to a certain degree of accuracy. This level of accuracy will have a bearing on the final result and is typical of what a GCSE question will want to find out.

Note the following as these tend to be the crux of GCSE questions :

When adding, the lower bound of the result is the sum of the lower limits of the two values. The upper bound is the sum of the upper limits of the two values. To assist us with the upper and lower bounds using subtraction this diagram will help:



For the biggest possible number

For the smallest possible number

When multiplying  
When dividing

biggest no. x biggest no.  
biggest no. ÷ smallest no.

smallest no. x smallest no.  
smallest no. ÷ biggest no.

Examples: For these examples the numbers have been given to 1 decimal place.

<u>Adding.</u>	usual answer		lower bound		upper bound
	3.7		3.65		3.75
	<u>4.6</u>	+	<u>4.55</u>	+	<u>4.65</u>
	8.3		8.20		8.40

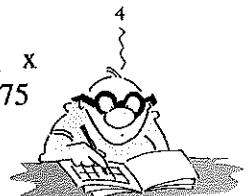
Therefore the answers accuracy lies between 8.20 and 8.40.

<u>Subtracting.</u>	usual answer		lower bound		upper bound
	7.7		7.65		7.75
	<u>4.6</u>	-	<u>4.65</u>	-	<u>4.55</u>
	3.1		3.00		3.20

Therefore the answers accuracy lies between 3.00 and 3.20.

<u>Multiplying.</u>	usual answer		lower bound		upper bound
	3.7		3.65		3.75
	<u>4.6</u>	x	<u>4.55</u>	x	<u>4.65</u>
	17.02		16.6075		17.4375

Therefore the answers accuracy lies between 16.6075 and 17.4375.



<u>Dividing.</u>	usual answer		lower bound		upper bound
	<u>5.5</u>	= 5	<u>5.45</u>	= 4.739	<u>5.55</u>
	1.1		1.15		1.05

Therefore the answers accuracy lies between 4.739 and 5.286.

## Accuracy of Measurement 1.



1). Give the lower and upper bounds of these numbers.

- |                                   |                                     |           |                                |          |
|-----------------------------------|-------------------------------------|-----------|--------------------------------|----------|
| a). 4.43                          | b). 0.54                            | c). 3.264 | d). 46                         | e). 3.04 |
| f). 150 (given to the nearest 10) | g). 6800 (given to the nearest 100) |           |                                |          |
| h). 21.0                          | i). 65                              | j). 0.07  | k). 3                          | l). 3.0  |
| m). 3.00                          | n). 45.00                           | o). 79    | p). 3400 (given to 2 sig.fig.) |          |
| q). 3400 (given to 3 sig.fig.)    | r). 4.98                            | s). 4567  |                                |          |

2). In the following questions the numbers given are all to the nearest whole number. Work out the answer and then the lower and upper bounds of the answers.

- |                   |                   |                    |                    |
|-------------------|-------------------|--------------------|--------------------|
| a). $65 + 78$     | b). $4 + 3$       | c). $7 + 32$       | d). $97 + 88$      |
| e). $98 - 46$     | f). $92 - 79$     | g). $9 - 2$        | h). $45 - 9$       |
| i). $3 \times 60$ | j). $92 \times 3$ | k). $50 \times 21$ | l). $49 \times 12$ |
| m). $48 \div 5$   | n). $54 \div 9$   | o). $69 \div 20$   | p). $312 \div 21$  |

3). In the following questions the numbers given are all to 2 significant figures. Work out the answer and then the lower and upper bounds of the answers.

- |                      |                      |                      |                      |
|----------------------|----------------------|----------------------|----------------------|
| a). $65 + 470$       | b). $45 + 370$       | c). $780 + 320$      | d). $490 + 88$       |
| e). $7.9 - 4.6$      | f). $24 - 7.9$       | g). $980 - 29$       | h). $9.0 - 6.7$      |
| i). $3.0 \times 6.4$ | j). $2.9 \times 3.0$ | k). $5.0 \times 2.9$ | l). $4.6 \times 6.4$ |
| m). $78 \div 5.0$    | n). $52 \div 9.4$    | o). $6900 \div 290$  | p). $3900 \div 92$   |

4). In the following questions the numbers given are all to 1 decimal place. Work out the answer and then the lower and upper bounds of the answers.

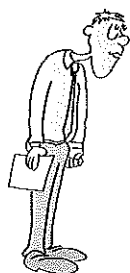
- |                      |                      |                      |                      |
|----------------------|----------------------|----------------------|----------------------|
| a). $2.5 + 4.7$      | b). $0.4 + 0.3$      | c). $0.8 + 3.2$      | d). $4.7 + 8.8$      |
| e). $7.8 - 4.6$      | f). $14.2 - 7.9$     | g). $3.1 - 2.3$      | h). $9.5 - 6.0$      |
| i). $3.0 \times 6.8$ | j). $2.2 \times 3.1$ | k). $5.0 \times 2.1$ | l). $4.7 \times 1.5$ |
| m). $4.8 \div 5.0$   | n). $5.8 \div 9.2$   | o). $6.9 \div 2.9$   | p). $3.7 \div 9.1$   |

5). In the following questions the numbers given are all to 2 decimal places. Work out the answer and then the lower and upper bounds of the answers.

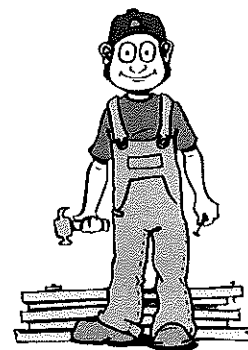
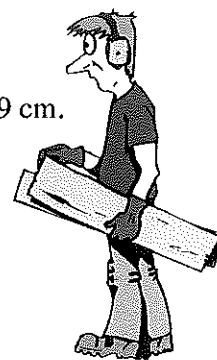
- |                        |                        |                        |                        |
|------------------------|------------------------|------------------------|------------------------|
| a). $2.65 + 4.78$      | b). $0.04 + 0.37$      | c). $0.78 + 3.26$      | d). $4.97 + 8.83$      |
| e). $7.98 - 4.61$      | f). $8.92 - 7.90$      | g). $3.91 - 2.03$      | h). $9.45 - 6.09$      |
| i). $3.02 \times 6.08$ | j). $2.92 \times 3.21$ | k). $5.00 \times 2.91$ | l). $4.97 \times 1.25$ |
| m). $4.78 \div 5.90$   | n). $5.28 \div 9.42$   | o). $6.92 \div 2.90$   | p). $3.71 \div 9.21$   |

6). In the following questions the first number is given to 3 significant figures and the second number is given to the nearest whole number. Work out the answer and then the lower and upper bounds of the answers.

- |                     |                    |                      |                     |
|---------------------|--------------------|----------------------|---------------------|
| a). $265 + 4$       | b). $0.04 + 3$     | c). $4780 + 32$      | d). $497 + 83$      |
| e). $798 - 461$     | f). $4920 - 79$    | g). $291 - 23$       | h). $90.4 - 6$      |
| i). $3.02 \times 6$ | j). $292 \times 2$ | k). $57800 \times 2$ | l). $497 \times 25$ |
| m). $478 \div 5$    | n). $5.28 \div 9$  | o). $69200 \div 290$ | p). $3710 \div 92$  |



- 7). Two sides of a rectangle were measured to the nearest mm, as 4.6 cm and 7.9 cm.  
 a). Find the least and greatest possible values of the perimeter.  
 b). Find the least and greatest possible values of the area.
- 8). Two sides of a rectangle were measured to the nearest m, as 12 m and 26 m.  
 a). Find the least and greatest possible values of the perimeter.  
 b). Find the least and greatest possible values of the area.
- 9). Two sides of a rectangle were measured to the nearest cm, as 4.51 m and 7.90 m.  
 a). Find the least and greatest possible values of the perimeter.  
 b). Find the least and greatest possible values of the area.
- 10). A rectangular field was measured to the nearest 10 m, as 1.23 km wide and 2.48 km long.  
 a). Find the least and greatest possible values of the perimeter.  
 b). Find the least and greatest possible values of the area.
- 11). A wooden rod was measured as 132.4 cm. After a length was cut off the new length was 63.2 cm. The measurements were made to the nearest mm.  
 What are the greatest and least values of the reduction in length ?
- 12). Two rods are measured and found to be 10.3 cm and 4.7 cm to the nearest mm.  
 What are the greatest and least values of the lengths of  
 a). each rod,  
 b). the two rods placed end to end ?
- 13). Bill measured the dimensions of his rectangular shaped garden. He found the length was 23.7 m and the width was 12.3 m, each measurement being correct to the nearest tenth of a metre.  
 a). Between what limits must the true length and width lie ?  
 b). Calculate the upper and lower limits for the area of his garden.
- 14). The weight of 1 cm<sup>3</sup> of silver is stated to be 10.6 g to 3 significant figures.  
 What are the lower and upper bounds for the weight of 30 cm<sup>3</sup> of silver ?
- 15). The weight of 1 cm<sup>3</sup> of gold is stated to be 19 g to 2 significant figures.  
 What are the lower and upper bounds for the weight of 80 cm<sup>3</sup> of gold ?
- 16). The weight of 1 cm<sup>3</sup> of petrol is stated to be 0.8 g to 1 decimal place.  
 What are the lower and upper bounds for the weight of 50 cm<sup>3</sup> of petrol ?
- 17). Four packages are weighed, each to the nearest gram.  
 Their weights are 453 g, 264 g, 836 g and 261 g.  
 What are the lower and upper bounds for the total weight ?
- 18). A square playground has a side length 45 m, to the nearest metre.  
 a). What are the lower and upper bounds for the perimeter ?  
 b). What are the lower and upper bounds for the area?





## Accuracy of Measurement 2.



### The Percentage Error.

The error is the difference between the estimated value and the actual true value. Therefore

$$\text{Percentage error} = \frac{\text{error}}{\text{true value}} \times 100$$

E.g. If you measure a line as 6.4 cm, then the upper bound is 6.45 and the lower bound is 6.35. As we do not know the actual true length we will use the measured length as an estimate. The maximum error possible = 0.05.

$$\text{The Maximum percentage error} = \frac{0.05}{6.4} \times 100 = 0.78 \% \text{ (2 d.p.)}$$

1). Find the percentage error for these measurements.

- a). 470 metres, measured to the nearest ten metres.
- b). 3.2 cm, measured to the nearest mm.
- c). 4600 mm measured to 2 significant figures.
- d). 3.45 m measured to 2 decimal places.
- e). 0.0345 measured to 3 significant figures.
- f). 350 cm measured to 2 significant figures.
- g). 23.4 mm measured to 1 decimal place.
- h). 15.3 m measured to 1 decimal place.
- i). 34000 m measured to the nearest thousand metres.
- j). 0.3 m measured to 1 decimal place.



2). Find the percentage error below.

- a). A measured length was 6.2 cm, the true length was calculated at 6.22 cm.
- b). A measured weight was 24.3 kg, the true weight was calculated at 24.24 kg.
- c). A measured volume was 2.0 m<sup>3</sup>, the true volume was calculated at 2.04 m<sup>3</sup>.
- d). A measured length was 43.2 m, the true length was calculated at 43.18 m.
- e).  $\pi$  was estimated at 3.14. A closer approximation is 3.14159.
- f).  $\sqrt{2}$  was estimated at 1.41. Use the value given on your calculator.

3). Two rods are measured and found to be 4.6 cm and 5.8 cm, to the nearest mm. The rods are placed end to end.

- a). What is the greatest length possible of the new rod ?
- b). What is the least possible length of the new rod ?
- c). What percentage error is possible ?

4). Four identical blocks are weighed to the nearest 100g. Each weighs 1.4 kg. The blocks are placed end to end.

- a). What is the greatest possible weight of the 4 blocks together ?
- b). What is the least possible weight of the 4 blocks together ?
- c). What percentage error is possible ?



## The Maximum Percentage Error.



The only estimate in the previous calculation was the 'true value'.

$$\text{Percentage error} = \frac{\text{error}}{\text{true value}} \times 100$$

To maximise this calculation we need to make the 'true value' as small as possible. So if we repeat the example on the last page.

E.g. If you measure a line as 6.4 cm, then the upper bound is 6.45 and the lower bound is 6.35. As we do not know the actual true length we will use the lower bound. The maximum error possible = 0.05.

$$\text{The Maximum percentage error} = \frac{0.05}{6.35} \times 100 = 0.79\% \text{ (2 d.p.)}$$

Hence

$\text{Maximum Percentage Error} = \frac{\text{largest possible error}}{\text{least possible value}} \times 100$
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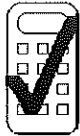
- 1). Find the maximum percentage error for these measurements.
  - a). 470 metres, measured to the nearest ten metres.
  - b). 3.2 cm, measured to the nearest mm.
  - c). 4600 mm measured to 2 significant figures.
  - d). 3.45 m measured to 2 decimal places.
  - e). 0.0345 measured to 3 significant figures.
  - f). 350 cm measured to 2 significant figures.
  - g). 23.4 mm measured to 1 decimal place.
  - h). 15.3 m measured to 1 decimal place.
  - i). 34000 m measured to the nearest thousand metres.
  - j). 0.3 m measured to 1 decimal place.



Compare these with your answers on the previous page.

- 2). The area of a rectangle is measured at 4.6 cm by 5.8 cm to the nearest mm.
  - a). Find the greatest possible area of the rectangle.
  - b). Find the smallest possible area of the rectangle.
  - c). Find the maximum percentage error possible.
- 3). The area of a rectangular playground is measured at 35.6 m by 18.7 m to 1 decimal place.
  - a). Find the greatest possible area of the playground.
  - b). Find the smallest possible area of the playground.
  - c). Find the maximum percentage error possible.
- 4). The length of one side of a large square is 3.4 m to the nearest 10 cm. Find the maximum possible percentage error of the **area** of the square.
- 5). The length of one side of a large square field is 2.3 km to 1 decimal place. Find the maximum possible percentage error of the **area** of the field.

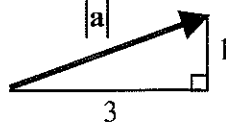




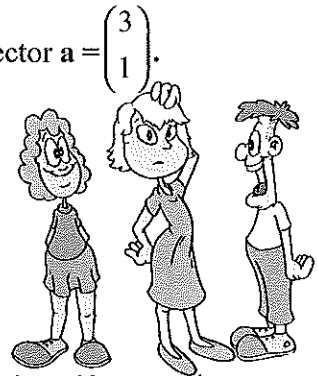
## Magnitude of a Vector

The magnitude, or modulus, of a vector is just a term for its length. The length of a column vector can be found using Pythagoras' theorem. For example, consider the vector  $\mathbf{a} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ .

The notation for the length of  $\mathbf{a}$  is  $|\mathbf{a}|$

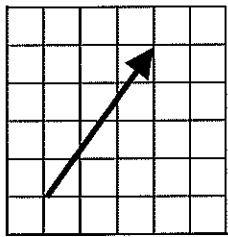


Using Pythagoras' theorem  $|\mathbf{a}|^2 = 3^2 + 1^2 \Rightarrow = \sqrt{10}$ .

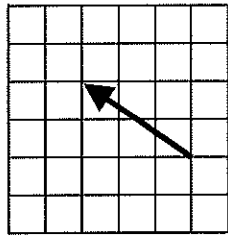


A. Find the magnitudes of the following vectors, leaving your answer in surd form if appropriate.

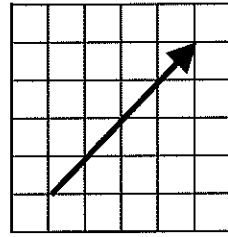
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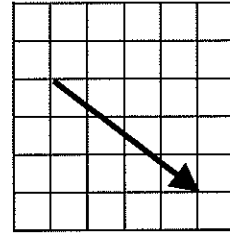
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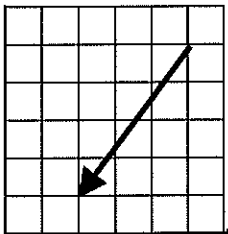
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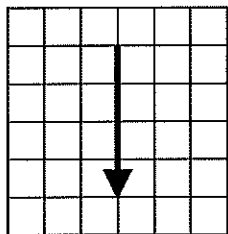
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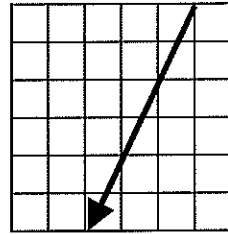
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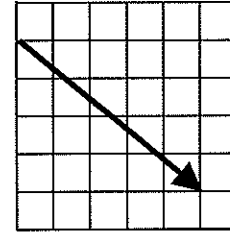
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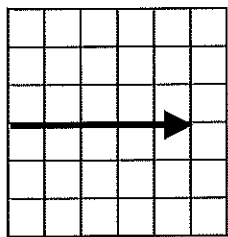
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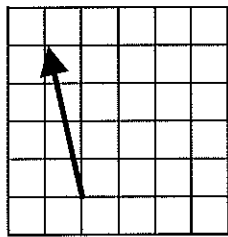
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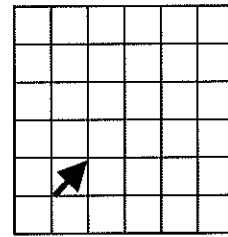
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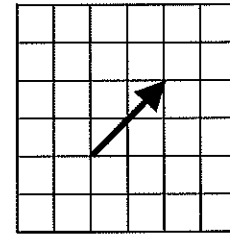
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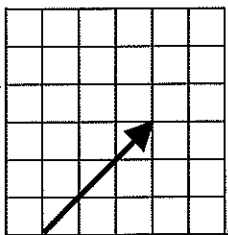
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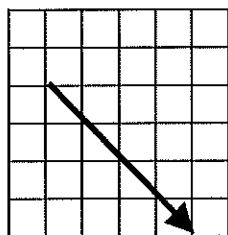
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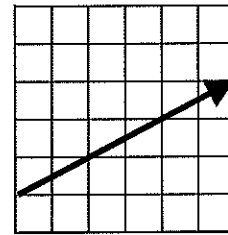
13.



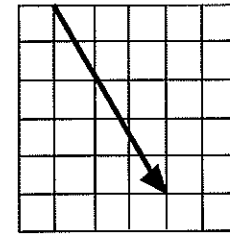
14.



15.



16.





It is useful if you can calculate the length of a vector without drawing a diagram. Pythagoras' theorem gives the following formula.

$$\begin{pmatrix} a \\ b \end{pmatrix} = \sqrt{a^2 + b^2}. \quad \text{E.g. } \begin{pmatrix} 4 \\ -2 \end{pmatrix} = \sqrt{4^2 + (-2)^2} = \sqrt{20} = 2\sqrt{5}$$

B. Find the lengths of the following vectors, leaving your answer in surd form if appropriate.

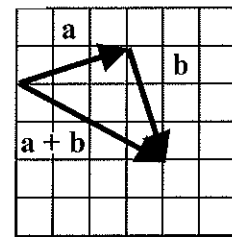
1.  $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$     2.  $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$     3.  $\begin{pmatrix} 5 \\ 12 \end{pmatrix}$     4.  $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$     5.  $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$     6.  $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$     7.  $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$
8.  $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$     9.  $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$     10.  $\begin{pmatrix} 7 \\ 24 \end{pmatrix}$     11.  $\begin{pmatrix} -7 \\ -1 \end{pmatrix}$     12.  $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$     13.  $\begin{pmatrix} 5 \\ -5 \end{pmatrix}$     14.  $\begin{pmatrix} 8 \\ -4 \end{pmatrix}$

C. The diagram shows three vectors,  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{a} + \mathbf{b}$ . Find the length of each vector and use Pythagoras's theorem to show that  $\mathbf{a}$  is perpendicular to  $\mathbf{b}$ .

**Solution:**

$$\mathbf{a} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \text{ and } \mathbf{a} + \mathbf{b} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}.$$

$$|\mathbf{a}| = \sqrt{10}, \quad |\mathbf{b}| = \sqrt{10}, \quad |\mathbf{a} + \mathbf{b}| = \sqrt{20}.$$



Using Pythagoras' theorem  $(\sqrt{10})^2 + (\sqrt{10})^2 = 10 + 10 = 20 = (\sqrt{20})^2$

therefore  $\mathbf{a}$  and  $\mathbf{b}$  are perpendicular.

1. In each of the following cases draw a diagram showing  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{a} + \mathbf{b}$ , find the length of each vector, and use Pythagoras' theorem to verify that  $\mathbf{a}$  is perpendicular to  $\mathbf{b}$ .

(i)  $\mathbf{a} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$     (ii)  $\mathbf{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$     (iii)  $\mathbf{a} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$

(iv)  $\mathbf{a} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$     (v)  $\mathbf{a} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$     (vi)  $\mathbf{a} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

(vii)  $\mathbf{a} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$     (viii)  $\mathbf{a} = \begin{pmatrix} 8 \\ -2 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} -2 \\ -8 \end{pmatrix}$     (ix)  $\mathbf{a} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

2. Use Pythagoras' theorem to prove that the vectors  $\begin{pmatrix} x \\ y \end{pmatrix}$  and  $\begin{pmatrix} -y \\ x \end{pmatrix}$  are perpendicular.

3. The diagram shows the two vectors  $\begin{pmatrix} a \\ b \end{pmatrix}$  and  $\begin{pmatrix} c \\ d \end{pmatrix}$ .

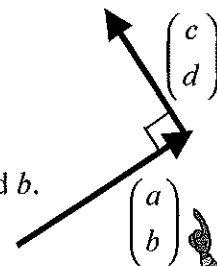
(a) Write down the sum of these vectors as a column vector.

(b) Write down, in surd form, the length of  $\begin{pmatrix} a \\ b \end{pmatrix}$ , in terms of  $a$  and  $b$ .

(c) Write down, in surd form, the length of  $\begin{pmatrix} c \\ d \end{pmatrix}$ , in terms of  $c$  and  $d$ .

(d) Write down, in surd form, the length of the sum of these vectors in terms of  $a$ ,  $b$ ,  $c$ , and  $d$ .

(e) If the vectors are perpendicular prove that  $ac + bd = 0$ .





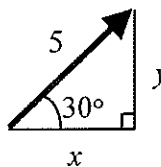
# Components

Sometimes we know the length and direction of a vector but need to know its column vector (component) form.

Find the following vectors as column vectors.

Give answers to one decimal place.

The vector of length 5 at  $30^\circ$  to the  $x$ -axis can be split into two components using trigonometry.

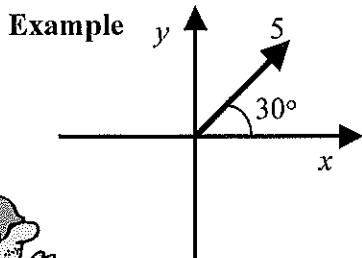


$$\cos 30^\circ = \frac{x}{5} \Rightarrow x = 5 \cos 30 = 4.3$$

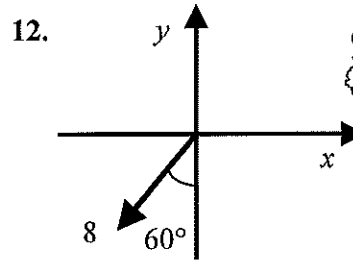
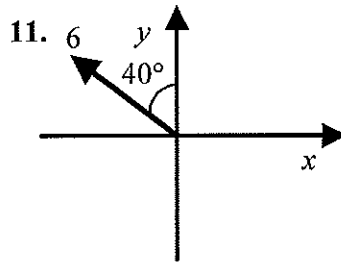
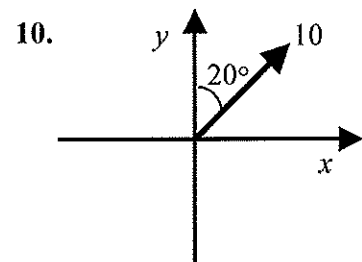
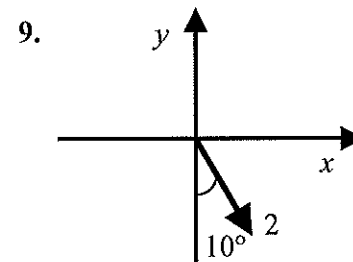
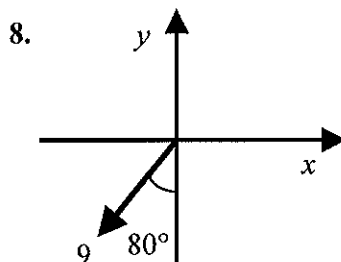
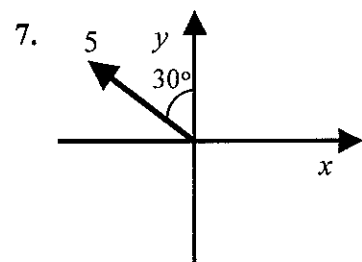
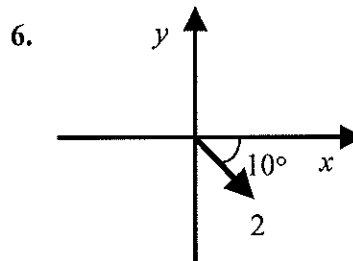
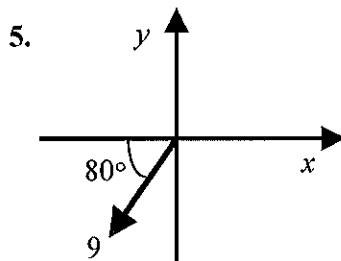
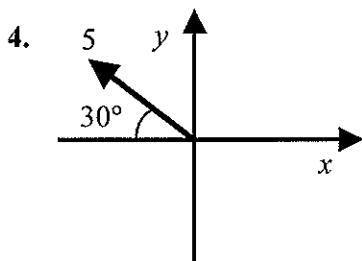
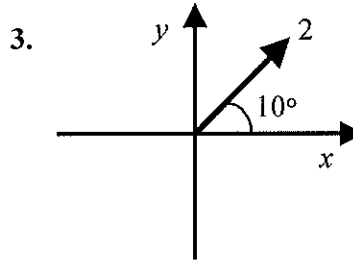
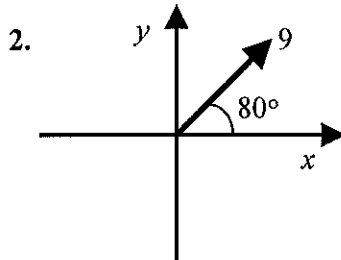
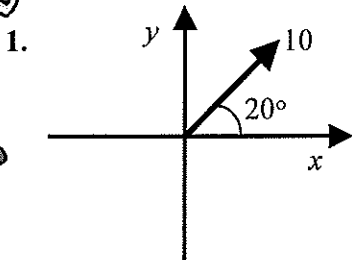
$$\sin 30^\circ = \frac{y}{5} \Rightarrow y = 5 \sin 30 = 2.5$$



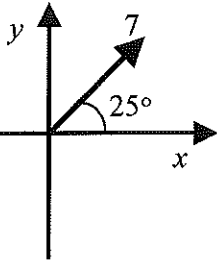
So the vector is  $\begin{pmatrix} 4.3 \\ 2.5 \end{pmatrix}$



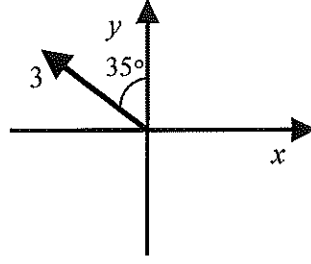
Diagrams not to scale



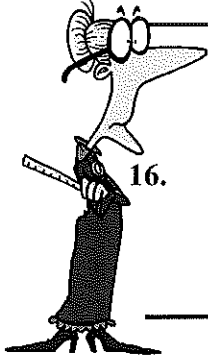
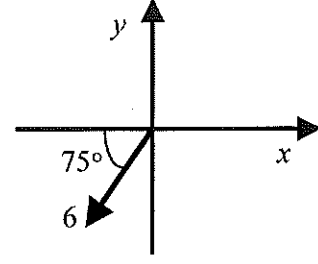
13.



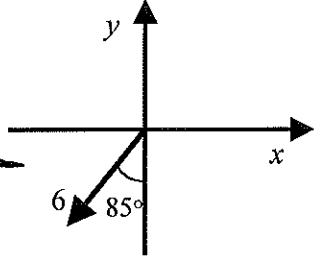
14.



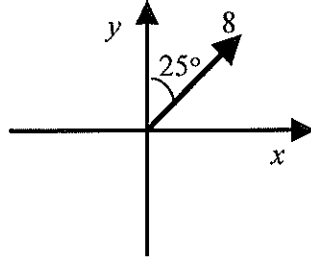
15.



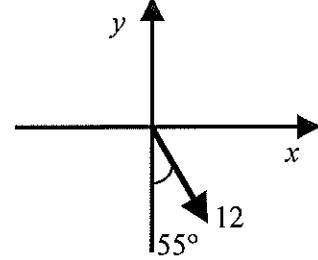
16.



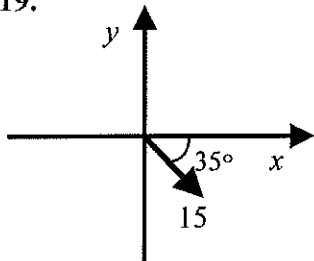
17.



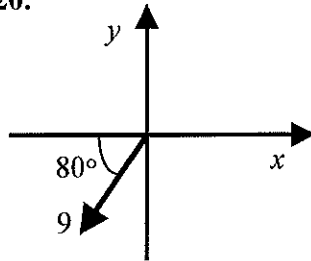
18.



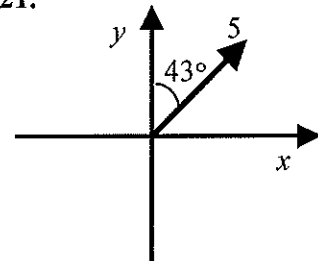
19.



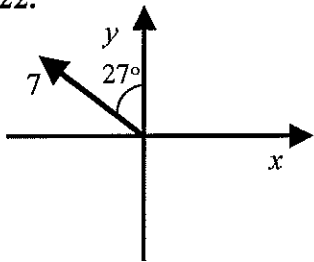
20.



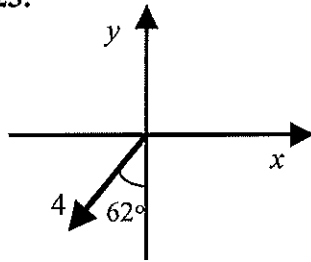
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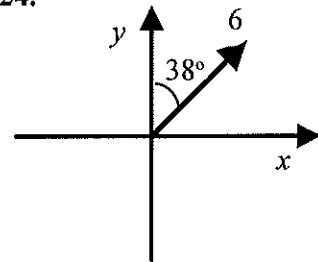
22.



23.

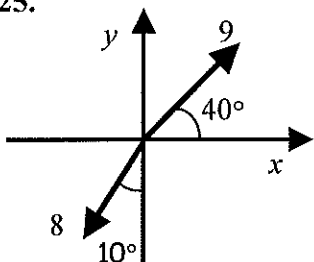


24.

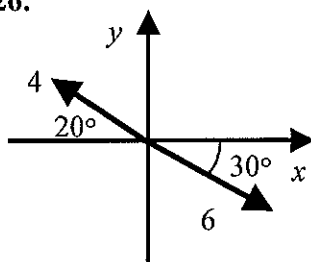


In the last six questions find each vector in column form and hence add the vectors together.

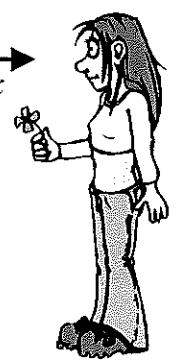
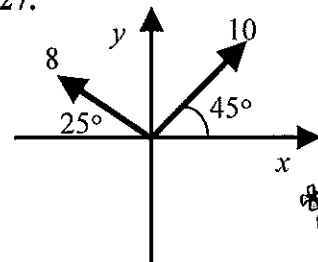
25.



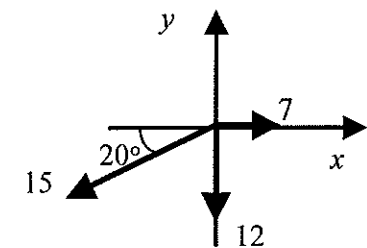
26.



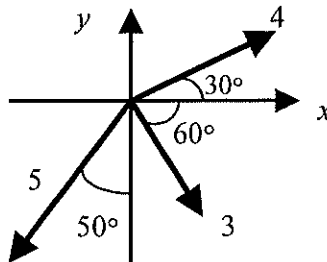
27.



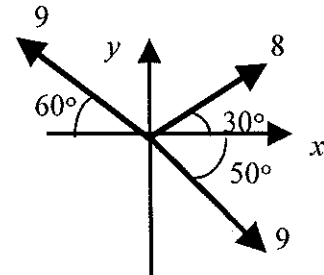
28.



29.



30.



## Level 9/10 Pack 1. Answers.

### Page 5. Accuracy of Measurement 1.

- 1). a). 4.425 4.435 b). 0.535 0.545 c). 3.2635 3.2645 d). 45.5 46.5  
 e). 3.035 3.045 f). 145 155 g). 6750 6850 h). 20.95 21.05  
 i). 64.5 65.5 j). 0.065 0.075 k). 2.5 3.5 l). 2.95 3.05  
 m). 2.995 3.005 n). 44.995 45.005 o). 78.5 79.5 p). 3350 3450  
 q). 3395 3405 r). 4.975 4.985 s). 4566.5 4567.5
- 2). a). 143 142 144 b). 7 6 8 c). 39 38 40 d). 185 184 186  
 e). 52 51 53 f). 13 12 14 g). 7 6 8 h). 36 35 37  
 i). 180 148.75 211.75 j). 276 228.75 323.75 k). 1050 1014.75 1085.75  
 l). 588 557.75 618.75 m). 9.6 8.6363 10.7778 n). 6 5.6316 6.4118  
 o). 3.45 3.3415 3.5641 p). 14.8571 14.4884 15.2439
- 3). a). 535 529.5 540.5 b). 415 409.5 420.5 c). 1100 1090 1110  
 d). 578 572.5 583.5 e). 3.3 3.2 3.4 f). 16.1 15.55 16.65  
 g). 951 945.5 956.5 h). 2.3 2.2 2.4 i). 19.2 18.7325 19.6725  
 j). 8.7 8.4075 8.9975 k). 14.5 14.1075 14.8975 l). 29.44 28.8925 29.9925  
 m). 15.6 14.0909 15.8586 n). 5.5319 5.4497 5.6150 o). 23.793 23.2203 24.3860  
 p). 42.3913 41.6216 43.1694
- 4). a). 7.2 7.1 7.3 b). 0.7 0.6 0.8 c). 4.0 3.9 4.1  
 d). 13.5 13.4 13.6 e). 3.2 3.1 3.3 f). 6.3 6.2 6.4  
 g). 0.8 0.7 0.9 h). 3.5 3.4 3.6 i). 20.4 19.9125 20.8925  
 j). 6.82 6.5575 7.0875 k). 10.5 10.1475 10.8575 l). 7.05 6.7425 7.3625  
 m). 0.96 0.9406 0.9798 n). 0.6304 0.6216 0.6393 o). 2.3793 2.3220 2.4386  
 p). 0.4066 0.3989 0.4144
- 5). a). 7.43 7.42 7.44 b). 0.41 0.40 0.42 c). 4.04 4.03 4.05  
 d). 13.8 13.79 13.81 e). 3.37 3.36 3.38 f). 1.02 1.01 1.03  
 g). 1.88 1.87 1.89 h). 3.36 3.35 3.37 i). 18.3616 18.3161 18.4071  
 j). 9.3732 9.3426 9.4039 k). 14.5500 14.5105 14.5896  
 l). 6.2125 6.1814 6.2436 m). 0.8102 0.8086 0.8117  
 n). 0.5605 0.5597 0.5613 o). 2.3862 2.3804 2.3921  
 p). 0.4028 0.4021 0.4036
- 6). a). 269 268 270 b). 3.04 2.535 3.545 c). 4812 4806.5 4817.5  
 d). 580 579 581 e). 337 336 338 f). 4841 4835.5 4846.5  
 g). 268 267 269 h). 84.4 83.85 84.95 i). 18.12 16.5825 19.6625  
 j). 584 437.25 731.25 k). 115600 86625 144625  
 l). 12425 12164 12686 m). 95.6 86.818 106.33  
 n). 0.5867 0.5553 0.6218 o). 238.6207 238.0379 239.2055  
 p). 40.3261 40.0541 40.6011

### Page 6.

- 7). a). 24.8 cm 25.2 cm b). 35.7175 cm<sup>2</sup> 36.9675 cm<sup>2</sup>  
 8). a). 74 m 78 m b). 293.25 m<sup>2</sup> 331.25 m<sup>2</sup>  
 9). a). 24.8 m 24.84 m b). 35.567 m<sup>2</sup> 35.6911 m<sup>2</sup>  
 10). a). 7.4 km 7.44 km b). 3.0319 km<sup>2</sup> 3.0690 km<sup>2</sup>  
 11). 69.3 cm 69.1 cm 12). a). 10.35/10.25 cm 4.75/4.65 cm b). 15.1 14.9cm  
 13). a). 12.25/12.35 and 23.65/23.75 m b). 293.3125 289.7125  
 14). 316.5 319.5 g 15). 1480 1560 g 16). 37.5 42.5 g  
 17). 1812 1816 g 18). a). 178 182 m b). 1980.25 2070.25 m<sup>2</sup>

### Page 7. Accuracy of Measurement 2.

- 1). a). 1.06% b). 1.565% c). 1.09% d). 0.14% e). 0.14%

- 2). f). 1.43% g). 0.21% h). 0.33% i). 1.47% j). 16.67%  
 a). 0.32% b). 0.25% c). 2.00% d). 0.05% e). 0.05%  
 f). 0.30%
- 3). a). 10.5 cm b). 10.3 cm c). 0.96%
- 4). a). 5.8 Kg b). 5.4 Kg c). 3.57%

**Page 8.**

- 1). a). 1.08% b). 1.59% c). 1.10% d). 0.15% e). 0.15%  
 f). 1.45% g). 0.21% h). 0.33% i). 1.49% j). 20.00%
- 2). a).  $27.2025\text{cm}^2$  b). 26.1625 c). 2.00%
- 3). a).  $668.4375\text{m}^2$  b). 663.0075 c). 0.41%
- 4). 3.05% 5). 4.59%

**Page 9. Accuracy of Measurement 3.**

- 1). Should be 14.0 cm if exact to nearest mm
- 2). a). 22.02 m b). 29.945025 m<sup>2</sup> 3). 364 days (or 365 if leap year)
- 4). 14.8 is measured to nearest 1/10 mm, 14.80 is measured to nearest 1/100 mm
- 5). a). 21.4 m b). 23.1525 m<sup>2</sup> 6). 1.4 m could be 135 - 145 cm.
- 7). a). 9.5 10.5 mm b). 52.25 68.25 mm<sup>2</sup> c). 8
- 8). a). 55.5 135.5 cm b). 7330.25 cm<sup>2</sup> c). 0.53%
- 9). a). 0.155 cm b). 0.045 cm
- 10). a). 17.5 18.5 mm b). 148.75 175.75 mm<sup>2</sup> c). 14
- 11). a). 58.5 192.5 b). 11011.25 c). 0.40%

**Page 10.**

- 12). a). 24.355 24.365 l b). 4.47 l
- 13). a). 11.65 11.55 s b). 100.05 99.95 m c). 8.66 m/s
- 14). 3.982 3.997 cm 15). 15.38% 22.22% 16). 5.49 5.75
- 17). 16.47% 21.69% 18). a). 56.05 56.15 b). 399.5 400.5 c). 7.15 m/s
- 19). 21.78% 26.26% 20). 8.25 8.40

**Page 11. Variation 1.**

Revision

- 1). a).  $y = kx$  b). 6 c).  $y = 6x$
- 2). a).  $P = kV$  b). 9.5 c).  $P = 9.5V$
- 3). 48 4). 28 5). a). 63 b). 10 6). a). 12 b). 24
- 7). a). 102.6 b). 22 8).  $y = 10, x = 3$  9).  $x = 60, y = 0.35$
- 10).  $y = 18, x = 2.5$  11).  $y = 2$  12).  $y = 6, x = 0.75$
- 13). a).  $c = 0.11n$  b). 2.09 euros c). 31
- 14). a). 95 km b). 3.25 hours 15). 46.8 cm 16). 28.98 euros

**Page 12.**

- 1). a).  $y = kx^2$  b). 4 c).  $y = 4x^2$
- 2). a).  $P = k\sqrt{V}$  b). 1.2 c).  $P = 1.2\sqrt{V}$
- 3). a).  $T = kS^3$  b). 8.5 c).  $T = 8.5S^3$
- 4). 125 5). 147, 2 (-2) 6). 6.25 7).  $r = 5, r = 4.5$  8). 24
- 9). 8000 10). 1.25 11). 11.2 12).  $y = 4, y = 2/3$  13). a). 64.8 b). 8
- 14). a). 16.2 b). 16 15). a). 33 b). 12 (-12)
- 16). a). 42 b). 4/9 17). a). 20 b). 3.8
- 18). a).  $2\frac{2}{3}$  b). 6 19).  $y = 3/4, x = 2$  20).  $y = 4, x = 8$
- 21).  $V = 820.125, r = 6$  22).  $y = 12, z = 9$  23).  $T = 3.36, l = 2.78$
- 24).  $w = 18.796 \text{ kg}$  25).  $w = 1296 \text{ g}$

**Page 13. Variation 2.**

- 1). a).  $y = k/x$  b). 9 c).  $y = 9/x$
- 2). a).  $P = k/V$  b). 38.7 c).  $P = 38.7/V$

**Page 7. Subtracting Column Vectors**

- A.** 1.  $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$  2.  $\begin{pmatrix} 2 \\ 6 \end{pmatrix}$  3.  $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$  4.  $\begin{pmatrix} -2 \\ 7 \end{pmatrix}$  5.  $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$  6.  $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$  7.  $\begin{pmatrix} -4 \\ -7 \end{pmatrix}$  8.  $\begin{pmatrix} -1 \\ -7 \end{pmatrix}$
9.  $\begin{pmatrix} 1 \\ 8 \end{pmatrix}$  10.  $\begin{pmatrix} 6 \\ -4 \end{pmatrix}$  11.  $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$  12.  $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$  13.  $\begin{pmatrix} 3 \\ -7 \end{pmatrix}$  14.  $\begin{pmatrix} -1 \\ -7 \end{pmatrix}$  15.  $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$
- B.** 1.  $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$  2.  $\begin{pmatrix} -2 \\ -6 \end{pmatrix}$  3.  $\begin{pmatrix} -3 \\ -1 \end{pmatrix}$  4.  $\begin{pmatrix} 2 \\ -7 \end{pmatrix}$  5.  $\begin{pmatrix} -4 \\ -4 \end{pmatrix}$  6.  $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$  7.  $\begin{pmatrix} 4 \\ 7 \end{pmatrix}$  8.  $\begin{pmatrix} 1 \\ 7 \end{pmatrix}$
9.  $\begin{pmatrix} -1 \\ -8 \end{pmatrix}$  10.  $\begin{pmatrix} -6 \\ 4 \end{pmatrix}$  11.  $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$  12.  $\begin{pmatrix} -6 \\ -3 \end{pmatrix}$  13.  $\begin{pmatrix} -3 \\ 7 \end{pmatrix}$  14.  $\begin{pmatrix} 1 \\ 7 \end{pmatrix}$  15.  $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$
- C.** 1.  $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$  2.  $\begin{pmatrix} -2 \\ -1 \end{pmatrix}$  3.  $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$  4.  $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$  5.  $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$  6.  $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$  7.  $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$  8.  $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
9.  $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$  10.  $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$  11.  $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$  12.  $\begin{pmatrix} -5 \\ -1 \end{pmatrix}$  13.  $\begin{pmatrix} -5 \\ 0 \end{pmatrix}$  14.  $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$  15.  $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$  16.  $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$
17.  $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$  18.  $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$  19.  $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$  20.  $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$  21.  $\begin{pmatrix} 1 \\ 6 \end{pmatrix}$  22.  $\begin{pmatrix} -5 \\ -1 \end{pmatrix}$  23.  $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$  24.  $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$
25.  $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$  26.  $\begin{pmatrix} -3 \\ -9 \end{pmatrix}$  27.  $\begin{pmatrix} 5 \\ 10 \end{pmatrix}$  28.  $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$  29.  $\begin{pmatrix} 11 \\ 10 \end{pmatrix}$  30.  $\begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$  31.  $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$  32.  $\begin{pmatrix} -2 \\ -2 \end{pmatrix}$
33.  $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$  34.  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$  35.  $\begin{pmatrix} -1\frac{1}{2} \\ 0 \end{pmatrix}$

**Page 8. Magnitude of a Vector**

- A.** 1. 5 2.  $\sqrt{13}$  3.  $4\sqrt{2}$  4. 5 5. 5 6. 4 7.  $3\sqrt{5}$
8.  $\sqrt{41}$  9. 5 10.  $\sqrt{17}$  11.  $\sqrt{2}$  12.  $2\sqrt{2}$  13.  $3\sqrt{2}$  14.  $4\sqrt{2}$
15.  $3\sqrt{5}$  16.  $\sqrt{34}$

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- B.** 1. 5 2. 5 3. 13 4.  $\sqrt{13}$  5.  $2\sqrt{5}$  6.  $\sqrt{2}$  7.  $\sqrt{2}$
8.  $\sqrt{5}$  9.  $\sqrt{5}$  10. 25 11.  $5\sqrt{2}$  12.  $\sqrt{5}$  13.  $5\sqrt{2}$  14.  $4\sqrt{5}$
- C.** 1. (i)  $\sqrt{2}, \sqrt{2}, 2$  (ii)  $\sqrt{5}, \sqrt{5}, \sqrt{10}$  (iii)  $\sqrt{13}, \sqrt{13}, \sqrt{26}$  (iv)  $\sqrt{20}, \sqrt{5}, 5$
- (v)  $\sqrt{17}, \sqrt{17}, \sqrt{34}$  (vi)  $3\sqrt{5}, \sqrt{5}, 5\sqrt{2}$  (vii)  $2\sqrt{2}, 2\sqrt{2}, 4$  (viii)  $2\sqrt{17}, 2\sqrt{17}, 2\sqrt{34}$
- (ix)  $4\sqrt{5}, \sqrt{5}, \sqrt{85}$

3. (a)  $\begin{pmatrix} a+c \\ b+d \end{pmatrix}$  (b)  $\sqrt{a^2+b^2}$  (c)  $\sqrt{c^2+d^2}$  (d)  $\sqrt{(a+c)^2+(b+d)^2}$

**Page 10. Vector Equations**

- A.** 1.  $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$  2.  $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$  3.  $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$  4.  $\begin{pmatrix} 6 \\ 4 \end{pmatrix}$  5.  $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$  6.  $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$  7.  $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$  8.  $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

9.  $\begin{pmatrix} 2 \\ -1.5 \end{pmatrix}$  10.  $\begin{pmatrix} 1 \\ 6 \end{pmatrix}$  11.  $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$  12.  $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$  13.  $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$  14.  $\begin{pmatrix} -8 \\ 1 \end{pmatrix}$  15.  $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$  16.  $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$   
 17.  $\begin{pmatrix} -8 \\ 20 \end{pmatrix}$  18.  $\begin{pmatrix} 1 \\ 2.5 \end{pmatrix}$  19.  $\begin{pmatrix} -2 \\ 13 \end{pmatrix}$  20.  $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$  21.  $\begin{pmatrix} \frac{1}{6} \\ 0 \end{pmatrix}$

B. 1.  $x = \begin{pmatrix} 1.5 \\ 1.5 \end{pmatrix}, y = \begin{pmatrix} -1.5 \\ 3.5 \end{pmatrix}$  2.  $x = \begin{pmatrix} -1 \\ 3 \end{pmatrix}, y = \begin{pmatrix} 5 \\ -5 \end{pmatrix}$  3.  $x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, y = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$   
 4.  $x = \begin{pmatrix} 6 \\ 6 \end{pmatrix}, y = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$  5.  $x = \begin{pmatrix} 1 \\ -3 \end{pmatrix}, y = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$  6.  $x = \begin{pmatrix} -0.5 \\ 1.5 \end{pmatrix}, y = \begin{pmatrix} -1.75 \\ 0.25 \end{pmatrix}$   
 7.  $x = \begin{pmatrix} 2 \\ -6 \end{pmatrix}, y = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$  8.  $x = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, y = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$  9.  $x = \begin{pmatrix} 0 \\ 2 \end{pmatrix}, y = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$   
 10.  $x = \begin{pmatrix} 0 \\ -6 \end{pmatrix}, y = \begin{pmatrix} 1 \\ -16 \end{pmatrix}$  11.  $x = \begin{pmatrix} -0.6 \\ -2.2 \end{pmatrix}, y = \begin{pmatrix} 1.4 \\ 1.8 \end{pmatrix}$  12.  $x = \begin{pmatrix} -5.5 \\ -8.5 \end{pmatrix}, y = \begin{pmatrix} -3.5 \\ -4.5 \end{pmatrix}$

Page 11. Components

1.  $\begin{pmatrix} 9.4 \\ 3.4 \end{pmatrix}$  2.  $\begin{pmatrix} 1.6 \\ 8.9 \end{pmatrix}$  3.  $\begin{pmatrix} 2.0 \\ 0.3 \end{pmatrix}$  4.  $\begin{pmatrix} -4.3 \\ 2.5 \end{pmatrix}$  5.  $\begin{pmatrix} -1.6 \\ -8.9 \end{pmatrix}$  6.  $\begin{pmatrix} 2.0 \\ -0.3 \end{pmatrix}$  7.  $\begin{pmatrix} -2.5 \\ 4.3 \end{pmatrix}$  8.  $\begin{pmatrix} -8.9 \\ -1.6 \end{pmatrix}$   
 9.  $\begin{pmatrix} 0.3 \\ -2.0 \end{pmatrix}$  10.  $\begin{pmatrix} 3.4 \\ 9.4 \end{pmatrix}$  11.  $\begin{pmatrix} -3.9 \\ 4.6 \end{pmatrix}$  12.  $\begin{pmatrix} -6.9 \\ -4 \end{pmatrix}$

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13.  $\begin{pmatrix} 6.3 \\ 3.0 \end{pmatrix}$  14.  $\begin{pmatrix} -1.7 \\ 2.5 \end{pmatrix}$  15.  $\begin{pmatrix} -1.6 \\ -5.8 \end{pmatrix}$  16.  $\begin{pmatrix} -6.0 \\ -0.5 \end{pmatrix}$  17.  $\begin{pmatrix} 3.4 \\ 7.3 \end{pmatrix}$  18.  $\begin{pmatrix} 9.8 \\ -6.9 \end{pmatrix}$  19.  $\begin{pmatrix} 12.3 \\ -8.6 \end{pmatrix}$  20.  $\begin{pmatrix} -1.6 \\ -8.9 \end{pmatrix}$   
 21.  $\begin{pmatrix} 3.4 \\ 3.7 \end{pmatrix}$  22.  $\begin{pmatrix} -3.2 \\ 6.2 \end{pmatrix}$  23.  $\begin{pmatrix} -3.5 \\ -1.9 \end{pmatrix}$  24.  $\begin{pmatrix} 3.7 \\ 4.7 \end{pmatrix}$  25.  $\begin{pmatrix} 5.5 \\ -2.1 \end{pmatrix}$  26.  $\begin{pmatrix} 1.4 \\ -1.6 \end{pmatrix}$  27.  $\begin{pmatrix} -0.2 \\ 10.5 \end{pmatrix}$  28.  $\begin{pmatrix} -7.1 \\ -17.1 \end{pmatrix}$   
 29.  $\begin{pmatrix} 1.2 \\ -3.8 \end{pmatrix}$  30.  $\begin{pmatrix} 8.2 \\ 4.9 \end{pmatrix}$

Page 13. General Vector Addition

1. (a) 5 (b) 6.8 (c) 7.9 (d) 3.6 (e) 4.6 (f) 4.9 (g) 0.7 (h) 4.6  
 2. (a) 7.1 (b) 6.8 (c) 7.8 (d) 4.6 (e) 8.3 (f) 6.5 (g) 1.3 (h) 6.6  
 3. (a) 7.1 (b) 2.1 (c) 4.6 (d) 7.8 (e) 3.6 (f) 5.1 (g) 9.0 (h) 7.8

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4. (a) 5 (b) 7 (c) 4.4 (d) 7.7 (e) 7.4 (f) 7.9 (g) 7.9 (h) 4.2  
 (i) 7.5 (j) 2.4  
 5. (a) 7.7 (b) 8.7 (c) 6.9 (d) 16.9 (e) 5 (f) 8.4 (g) 1.7 (h) 3.6  
 (i) 7.8 (j) 9.3 (k) 7.4 (l) 7.4 or 2.6  
 6. (a)  $2,090^\circ$  (b)  $6,000^\circ$  (c)  $4,135^\circ$  (d)  $3,270^\circ$  (e)  $1,315^\circ$  (f)  $2,2027^\circ$   
 (g)  $2,2,153^\circ$  (h)  $2,8,045^\circ$  (i)  $2,8,135^\circ$  (j)  $2,2,333^\circ$  (k)  $1,180^\circ$   
 (l)  $1,8,113^\circ$  (m)  $1,5,029^\circ$  (n)  $1,5,061^\circ$  (o)  $2,8,105^\circ$  (p)  $2,9,029^\circ$   
 (q)  $1,5,068^\circ$  (r)  $1,5,164^\circ$  (s)  $0,74,164^\circ$  (t)  $1,4,105^\circ$  (u)  $0,77,248^\circ$   
 (v)  $2,2,207^\circ$  (w)  $3,6,056^\circ$  (x)  $6,1,009^\circ$