SUMMER WORK FURTHER MATHEMATICS

Head of Department

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Exam Board

Edexcel

Specification

AS Level Mathematics
8MAO
AS Level Further Mathematics
8FMO

A Level Mathematics
9MAO
A Level Further Mathematics
9FMO

COURSE DETAILS

Examination
The AS Level Mathematics and Further Mathematics courses are examined at the end of Year 12, and the A Level Mathematics and Further Mathematics courses are examined at the end of Year 13.

AS Level Mathematics:
Pure Mathematics and Applied Mathematics:
Students will study elements of Pure Mathematics (proof, algebra and functions, coordinate geometry, sequences and series, trigonometry, exponentials and logarithms, differentiation, integration and vectors), elements of Statistics (statistical sampling, data presentation and interpretation, probability, statistical distributions and statistical hypothesis testing) and elements of Mechanics (quantities and units, kinematics, forces and Newton's laws).

AS Level Further Mathematics
Pure Mathematics and Applied Mathematics:
Students will study the compulsory elements of Further Pure Mathematics (proof, complex numbers, matrices, further algebra and functions, further calculus and further vectors). In addition to this, students will study two additional elements on Further Mechanics (momentum and impulse, work, energy and power and collisions in one dimension) and on Decision Mathematics (algorithms, graph theory, algorithms on graphs, critical path analysis and linear programming).

SUMMER WORK FOR
INTRODUCTION TO YEAR 12

	Task	Description
1.	Essential Work Prior to A Level Further Mathematics	**Complete the 'Essential Work prior to A Level Further Mathematics' over the Summer holidays.** **Guidance:** *1.* Read each question carefully. *2.* Attempt every question. *3.* Check and mark your answers. *4.* Always show your workings.
2.	Further Revision	**Use the following websites to consolidate any areas you struggled with when completing the Summer work** https://www.mymaths.co.uk/ (username: gordons password: angle) https://www.bbc.com/education/examspecs/z9p3mnb https://www.examsolutions.net/gcse-maths/ https://corbettmaths.com http://furthermaths.org.uk/gcse

WIDER READING TO PREPARE FOR COURSE

- A Mathematician's Apology by G.H. Hardy (CUP, 1992)
- Fermat's Last Theorem by Simon Singh
- The Music of the Primes by Marcus du Sautoy (Harper-Collins, 2003)
- Mathematics: a very short introduction by Timothy Gowers (CUP, 2002)
- Archimedes' Revenge by P. Hoffman (Penguin, 1991)
- Surely You're Joking, Mr. Feynman by R.P. Feynman (Arrow Books, 1992)
- Solving Mathematical Problems by Terence Tao (OUP, 2006)
- The Pleasures of Counting by T.W. Körner (CUP, 1996)

Gordon's School

Mathematics Department

Essential Work prior to A Level Further Mathematics

Name:__________________________________

Prior Work prior to A Level Further Mathematics

Workbook

A Level Mathematics and A Level Further Mathematics take as a base your knowledge of topics from GCSE. The following topics will be tested on arrival in September. Very few lessons will be spent going over these basic principles.

Time should be spent over the summer months preparing yourself for the preliminary test on the first week back.

Guidance

5. Complete the whole booklet.
6. Read each question carefully.
7. Attempt every question.
8. Check and mark your answers.
9. Always show your workings.
10. Spend additional time on topics you struggle with.

Algebra

1. Expand single, double and triple brackets
2. Factorise linear, quadratic and simple cubic expressions
3. Evaluate using laws of indices
4. Simplify and use the rules of surds
5. Rationalise the denominator
6. Solve quadratics using factorising, the quadratic formula and completing the square
7. Solve linear simultaneous equations
8. Solve quadratic simultaneous equations
9. Solve linear inequalities
10. Solve quadratic inequalities

Coordinate Geometry

11. Find the gradient of a line given two points
12. Understand the equation of a straight line
13. Find the equation of a line
14. Know rules for parallel and perpendicular lines

Trigonometry

15. The sine rule
16. The cosine rule
17. Mixed questions
18. Area of triangles

Further Algebra

19. Inequalities on Graphs
20. Transforming Graphs
21. Algebraic Fractions

Algebra

1. Expand single, double and triple brackets

1. Show that

   a $(3x+1)(x^2-2x+4) \equiv 3x^3-5x^2+10x+4$

   b $(1+2x-x^2)(1-2x+x^2) \equiv 1-4x^2+4x^3-x^4$

   c $(3-x)^3 \equiv 27-27x+9x^2-x^3$

2. Giving your answers in descending powers of x, expand and simplify

   a $(x+1)(x^2+5x-6)$ **b** $(2x-5)(x^2-3x+7)$

   c $(4-7x)(2+5x-x^2)$ **d** $(3x-2)^3$

   e $(x^2+3)(2x^2-x+9)$ **f** $(4x-1)(x^4-3x^2+5x+2)$

   g $(x^2+2x+5)(x^2+3x+1)$ **h** $(x^2+x-3)(2x^2-x+4)$

   i $(3x^2-5x+2)(2x^2-4x-8)$ **j** $(x^2+2x-6)^2$

   k $(x^3+4x^2+1)(2x^4+x^2+3)$ **l** $(6-2x+x^3)(3+x^2-x^3+2x^4)$

3. Simplify

   a $(p^2-1)(p+4)(2p+3)$

   b $(t+2)(t^2+3t+5)+(t+4)(t^2+t+7)$

   c $2(x^2-3)(x^2+x-4)+(3x-1)(4x^3+2x^2-x+6)$

   d $(u^3-4u^2-3)(u+2)-(2u^3+u-1)(u^2+5u-3)$

2. Factorise linear, quadratic and simple cubic expressions

1. Factorise

   a. $2x+6$ b x^2+6x c $6y^2-9y$ d $24x^2y^3z+15xy^2z^4$

2. Factorise

   a x^2+4x+3 **b** $x^2+7x+10$ **c** y^2-3y+2 **d** x^2-6x+9

   e y^2-y-2 **f** a^2+2a-8 **g** x^2-1 **h** $p^2+9p+14$

   i $x^2-2x-15$ **j** $16-10m+m^2$ **k** $t^2+3t-18$ **l** $y^2-13y+40$

   m r^2-16 **n** $y^2-2y-63$ **o** $121+22a+a^2$ **p** $x^2+6x-72$

   q $26-15x+x^2$ **r** $s^2+23s+120$ **s** $p^2+14p-51$ **t** m^2-m-90

3. Factorise

   a $2x^2+3x+1$ **b** $2+7p+3p^2$ **c** $2y^2-5y+3$ **d** $2-m-m^2$

   e $3r^2-2r-1$ **f** $5-19y-4y^2$ **g** $4-13a+3a^2$ **h** $5x^2-8x-4$

   i $4x^2+8x+3$ **j** $9s^2-6s+1$ **k** $4m^2-25$ **l** $2-y-6y^2$

   m $4u^2+17u+4$ **n** $6p^2+5p-4$ **o** $8x^2+19x+6$ **p** $12r^2+8r-15$

4. Factorise completely

   a x^3+7x^2+12x b $2x^3-7x^2-4x$ c $12x^3-5x^2-2x$

3. Using laws of indices

1. Evaluate

a 3^{-2} **b** $(\frac{2}{5})^{0}$ **c** $(-2)^{-6}$ **d** $(\frac{1}{6})^{-2}$ **e** $(1\frac{1}{2})^{-3}$ **f** $9^{\frac{1}{2}}$

g $16^{\frac{1}{4}}$ **h** $(-27)^{\frac{1}{3}}$ **i** $(\frac{1}{49})^{\frac{1}{2}}$ **j** $125^{\frac{1}{3}}$ **k** $(\frac{4}{9})^{\frac{1}{2}}$ **l** $36^{-\frac{1}{2}}$

m $81^{-\frac{1}{4}}$ **n** $(-64)^{-\frac{1}{3}}$ **o** $(\frac{1}{32})^{-\frac{1}{5}}$ **p** $(-\frac{8}{125})^{\frac{1}{3}}$ **q** $(2\frac{1}{4})^{\frac{1}{2}}$ **r** $(3\frac{3}{8})^{-\frac{1}{3}}$

2. Evaluate

a $4^{\frac{3}{2}}$ **b** $27^{\frac{2}{3}}$ **c** $16^{\frac{3}{4}}$ **d** $(-125)^{\frac{2}{3}}$ **e** $9^{\frac{5}{2}}$ **f** $8^{-\frac{2}{3}}$

g $36^{-\frac{3}{2}}$ **h** $(\frac{1}{8})^{\frac{4}{3}}$ **i** $(\frac{4}{9})^{\frac{3}{2}}$ **j** $(\frac{1}{216})^{-\frac{2}{3}}$ **k** $(\frac{9}{16})^{-\frac{3}{2}}$ **l** $(-\frac{27}{64})^{\frac{4}{3}}$

m $(0.04)^{\frac{1}{2}}$ **n** $(2.25)^{-\frac{3}{2}}$ **o** $(0.064)^{\frac{2}{3}}$ **p** $(1\frac{9}{16})^{-\frac{3}{2}}$ **q** $(5\frac{1}{16})^{\frac{3}{4}}$ **r** $(2\frac{10}{27})^{-\frac{4}{3}}$

3. Work out

a $4^{\frac{1}{2}} \times 27^{\frac{1}{3}}$ **b** $16^{\frac{1}{4}} + 25^{\frac{1}{2}}$ **c** $8^{-\frac{1}{3}} \div 36^{\frac{1}{2}}$ **d** $(-64)^{\frac{1}{3}} \times 9^{\frac{3}{2}}$

e $(\frac{1}{3})^{-2} - (-8)^{\frac{1}{3}}$ **f** $(\frac{1}{25})^{\frac{1}{2}} \times (\frac{1}{4})^{-2}$ **g** $81^{\frac{3}{4}} - (\frac{1}{49})^{-\frac{1}{2}}$ **h** $(\frac{1}{27})^{-\frac{1}{3}} \times (\frac{4}{9})^{-\frac{1}{2}}$

i $(\frac{1}{9})^{-\frac{1}{2}} \times (-32)^{\frac{3}{5}}$ **j** $(121)^{0.5} + (32)^{0.2}$ **k** $(100)^{0.5} \div (0.25)^{1.5}$ **l** $(16)^{-0.25} \times (243)^{0.4}$

4. Simplify

a $x^{8} \times x^{-6}$ **b** $y^{-2} \times y^{-4}$ **c** $6p^{3} \div 2p^{7}$ **d** $(2x^{-4})^{3}$

e $y^{3} \times y^{-\frac{1}{2}}$ **f** $2b^{\frac{2}{3}} \times 4b^{\frac{1}{4}}$ **g** $x^{\frac{2}{3}} \div x^{\frac{1}{3}}$ **h** $a^{\frac{1}{2}} \div a^{\frac{4}{3}}$

i $p^{\frac{1}{4}} \div p^{-\frac{1}{3}}$ **j** $(3x^{\frac{2}{3}})^{2}$ **k** $y \times y^{\frac{5}{6}} \times y^{-\frac{1}{2}}$ **l** $4t^{\frac{3}{2}} \div 12t^{\frac{1}{2}}$

m $\frac{b^{2} \times b^{\frac{1}{4}}}{b^{\frac{1}{2}}}$ **n** $\frac{y^{\frac{1}{2}} \times y^{\frac{1}{3}}}{y}$ **o** $\frac{4x^{\frac{2}{3}} \times 3x^{-\frac{1}{6}}}{6x^{\frac{3}{4}}}$ **p** $\frac{2a \times a^{\frac{3}{4}}}{8a^{-\frac{1}{2}}}$

4. Simplify and use the rules of surds

1. Simplify

a $\sqrt{12}$ **b** $\sqrt{28}$ **c** $\sqrt{80}$ **d** $\sqrt{27}$ **e** $\sqrt{24}$ **f** $\sqrt{128}$

g $\sqrt{45}$ **h** $\sqrt{40}$ **i** $\sqrt{75}$ **j** $\sqrt{112}$ **k** $\sqrt{99}$ **l** $\sqrt{147}$

m $\sqrt{216}$ **n** $\sqrt{800}$ **o** $\sqrt{180}$ **p** $\sqrt{60}$ **q** $\sqrt{363}$ **r** $\sqrt{208}$

2. Simplify

a $\sqrt{18} + \sqrt{50}$ **b** $\sqrt{48} - \sqrt{27}$ **c** $2\sqrt{8} + \sqrt{72}$

d $\sqrt{360} - 2\sqrt{40}$ **e** $2\sqrt{5} - \sqrt{45} + 3\sqrt{20}$ **f** $\sqrt{24} + \sqrt{150} - 2\sqrt{96}$

3. Express in the form $a + b\sqrt{3}$

a $\sqrt{3}(2 + \sqrt{3})$ **b** $4 - \sqrt{3} - 2(1 - \sqrt{3})$ **c** $(1 + \sqrt{3})(2 + \sqrt{3})$

d $(4 + \sqrt{3})(1 + 2\sqrt{3})$ **e** $(3\sqrt{3} - 4)^{2}$ **f** $(3\sqrt{3} + 1)(2 - 5\sqrt{3})$

4. Simplify

a $(\sqrt{5} + 1)(2\sqrt{5} + 3)$ **b** $(1 - \sqrt{2})(4\sqrt{2} - 3)$ **c** $(2\sqrt{7} + 3)^{2}$

d $(3\sqrt{2} - 1)(2\sqrt{2} + 5)$ **e** $(\sqrt{5} - \sqrt{2})(\sqrt{5} + 2\sqrt{2})$ **f** $(3 - \sqrt{8})(4 + \sqrt{2})$

5. Rationalise the denominator

1. Express each of the following as simply as possible with a rational denominator.

a $\frac{1}{\sqrt{5}}$ **b** $\frac{2}{\sqrt{3}}$ **c** $\frac{1}{\sqrt{8}}$ **d** $\frac{14}{\sqrt{7}}$ **e** $\frac{3\sqrt{2}}{\sqrt{3}}$ **f** $\frac{\sqrt{5}}{\sqrt{15}}$

g $\frac{1}{3\sqrt{7}}$ **h** $\frac{12}{\sqrt{72}}$ **i** $\frac{1}{\sqrt{80}}$ **j** $\frac{3}{2\sqrt{54}}$ **k** $\frac{4\sqrt{20}}{3\sqrt{18}}$ **l** $\frac{3\sqrt{175}}{2\sqrt{27}}$

2. Express each of the following as simply as possible with a rational denominator.

a $\frac{1}{\sqrt{2}+1}$ **b** $\frac{4}{\sqrt{3}-1}$ **c** $\frac{1}{\sqrt{6}-2}$ **d** $\frac{3}{2+\sqrt{3}}$

e $\frac{1}{2+\sqrt{5}}$ **f** $\frac{\sqrt{2}}{\sqrt{2}-1}$ **g** $\frac{6}{\sqrt{7}+3}$ **h** $\frac{1}{3+2\sqrt{2}}$

i $\frac{1}{4-2\sqrt{3}}$ **j** $\frac{3}{3\sqrt{2}+4}$ **k** $\frac{2\sqrt{3}}{7-4\sqrt{3}}$ **l** $\frac{6}{\sqrt{5}-\sqrt{3}}$

6. Solve quadratics using factorising, the quadratic formula and completing the square

Factorising

1. Using factorisation, solve each equation.

a $x^2 - 4x + 3 = 0$ **b** $x^2 + 6x + 8 = 0$ **c** $x^2 + 4x - 5 = 0$ **d** $x^2 - 7x = 8$

e $x^2 - 25 = 0$ **f** $x(x - 1) = 42$ **g** $x^2 = 3x$ **h** $27 + 12x + x^2 = 0$

i $60 - 4x - x^2 = 0$ **j** $5x + 14 = x^2$ **k** $2x^2 - 3x + 1 = 0$ **l** $x(x - 1) = 6(x - 2)$

m $3x^2 + 11x = 4$ **n** $x(2x - 3) = 5$ **o** $6 + 23x - 4x^2 = 0$ **p** $6x^2 + 10 = 19x$

q $4x^2 + 4x + 1 = 0$ **r** $3(x^2 + 4) = 13x$ **s** $(2x + 5)^2 = 5 - x$ **t** $3x(2x - 7) = 2(7x + 3)$

Quadratic Formula

2 Use the quadratic formula to solve each equation, giving your answers as simply as possible in terms of surds where appropriate.

a $x^2 + 4x + 1 = 0$ **b** $4 + 8t - t^2 = 0$ **c** $y^2 - 20y + 91 = 0$ **d** $r^2 + 2r - 7 = 0$

e $6 + 18a + a^2 = 0$ **f** $m(m - 5) = 5$ **g** $x^2 + 11x + 27 = 0$ **h** $2u^2 + 6u + 3 = 0$

i $5 - y - y^2 = 0$ **j** $2x^2 - 3x = 2$ **k** $3p^2 + 7p + 1 = 0$ **l** $t^2 - 14t = 14$

m $0.1r^2 + 1.4r = 0.9$ **n** $6u^2 + 4u = 1$ **o** $\frac{1}{2}y^2 - 3y = \frac{2}{3}$ **p** $4x(x - 3) = 11 - 4x$

Completing the square

3 Solve each equation by completing the square, giving your answers as simply as possible in terms of surds where appropriate.

a $y^2 - 4y + 2 = 0$ **b** $p^2 + 2p - 2 = 0$ **c** $x^2 - 6x + 4 = 0$ **d** $7 + 10r + r^2 = 0$

e $x^2 - 2x = 11$ **f** $a^2 - 12a - 18 = 0$ **g** $m^2 - 3m + 1 = 0$ **h** $9 - 7t + t^2 = 0$

i $u^2 + 7u = 44$ **j** $2y^2 - 4y + 1 = 0$ **k** $3p^2 + 18p = -23$ **l** $2x^2 + 12x = 9$

m $-m^2 + m + 1 = 0$ **n** $4x^2 + 49 = 28x$ **o** $1 - t - 3t^2 = 0$ **p** $2a^2 - 7a + 4 = 0$

7. Solve linear simultaneous equations

1 Solve each pair of simultaneous equations.

a $y = 3x$
$y = 2x + 1$

b $y = x - 6$
$y = \frac{1}{2}x - 4$

c $y = 2x + 6$
$y = 3 - 4x$

d $x + y - 3 = 0$
$x + 2y + 1 = 0$

e $x + 2y + 11 = 0$
$2x - 3y + 1 = 0$

f $3x + 3y + 4 = 0$
$5x - 2y - 5 = 0$

8. Solve quadratic simultaneous equations

1. Solve each pair of simultaneous equations.

a $x^2 - y + 3 = 0$
$x - y + 5 = 0$

b $2x^2 - y - 8x = 0$
$x + y + 3 = 0$

c $x^2 + y^2 = 25$
$2x - y = 5$

d $x^2 + 2xy + 15 = 0$
$2x - y + 10 = 0$

e $x^2 - 2xy - y^2 = 7$
$x + y = 1$

f $3x^2 - x - y^2 = 0$
$x + y - 1 = 0$

g $2x^2 + xy + y^2 = 22$
$x + y = 4$

h $x^2 - 4y - y^2 = 0$
$x - 2y = 0$

i $x^2 + xy = 4$
$3x + 2y = 6$

j $2x^2 + y - y^2 = 8$
$2x - y = 3$

k $x^2 - xy + y^2 = 13$
$2x - y = 7$

l $x^2 - 5x + y^2 = 0$
$3x + y = 5$

9. Solve linear inequalities

1 Find the set of values of x for which

a $2x + 1 < 7$ **b** $3x - 1 \geq 20$ **c** $2x - 5 > 3$ **d** $6 + 3x \leq 42$

e $5x + 17 \geq 2$ **f** $\frac{1}{3}x + 7 < 8$ **g** $9x - 4 \geq 50$ **h** $3x + 11 < 7$

i $18 - x > 4$ **j** $10 + 4x \leq 0$ **k** $12 - 3x < 10$ **l** $9 - \frac{1}{2}x \geq 4$

2 Solve each inequality.

a $2y - 3 > y + 4$ **b** $5p + 1 \leq p + 3$ **c** $x - 2 < 3x - 8$

d $a + 11 \geq 15 - a$ **e** $17 - 2u < 2 + u$ **f** $5 - b \geq 14 - 3b$

g $4x + 23 < x + 5$ **h** $12 + 3y \geq 2y - 1$ **i** $16 - 3p \leq 36 + p$

j $5(r - 2) > 30$ **k** $3(1 - 2t) \leq t - 4$ **l** $2(3 + x) \geq 4(6 - x)$

m $7(y + 3) - 2(3y - 1) < 0$ **n** $4(5 - 2x) > 3(7 - 2x)$ **o** $3(4u - 1) - 5(u - 3) < 9$

10. Solve quadratic inequalities

1. Find the set of values of x for which

a $x^2 - 4x + 3 < 0$ **b** $x^2 - 4 \leq 0$ **c** $15 + 8x + x^2 < 0$ **d** $x^2 + 2x \leq 8$

e $x^2 - 6x + 5 > 0$ **f** $x^2 + 4x > 12$ **g** $x^2 + 10x + 21 \geq 0$ **h** $22 + 9x - x^2 > 0$

i $63 - 2x - x^2 \leq 0$ **j** $x^2 + 11x + 30 > 0$ **k** $30 + 7x - x^2 > 0$ **l** $x^2 + 91 \geq 20x$

2. Solve each inequality.

a $2x^2 - 9x + 4 \leq 0$ **b** $2r^2 - 5r - 3 < 0$ **c** $2 - p - 3p^2 \geq 0$

d $2y^2 + 9y - 5 > 0$ **e** $4m^2 + 13m + 3 < 0$ **f** $9x - 2x^2 \leq 10$

Coordinate Geometry

11. Find the gradient of a line given two points

1 Find the gradient of the line segment joining each pair of points.

a (3, 1) and (5, 5) **b** (4, 7) and (10, 9) **c** (6, 1) and (2, 5) **d** (−2, 2) and (2, 8)

e (1, 3) and (7, −1) **f** (4, 5) and (−5, −7) **g** (−2, 0) and (0, −8) **h** (8, 6) and (−7, −2)

12. Understand the equation of a straight line

1. Write down the gradient and y-intercept of each line.

a $y = 4x - 1$ **b** $y = \frac{1}{3}x + 3$ **c** $y = 6 - x$ **d** $y = -2x - \frac{3}{5}$

2. Find the gradient and y-intercept of each line.

a $x + y + 3 = 0$ **b** $x - 2y - 6 = 0$ **c** $3x + 3y - 2 = 0$ **d** $4x - 5y + 1 = 0$

13. Find the equation of a line

1. Find, in the form $y = mx + c$, the equation of the straight line with the given gradient which passes through the given point.

a gradient 3, point (1, 2) **b** gradient −1, point (5, 3)

c gradient 4, point (−2, −3) **d** gradient −2, point (−4, 1)

e gradient $\frac{1}{3}$, point (−3, 1) **f** gradient $-\frac{5}{6}$, point (9, −2)

2. Find, in the form $y = mx + c$, the equation of the straight line passing through each pair of points.

a (0, 1) and (4, 13) **b** (2, 9) and (7, −1) **c** (−4, 3) and (2, 7)

d $(-\frac{1}{2}, -2)$ and (2, 8) **e** (3, −2) and (18, −5) **f** (−3.2, 4) and (−2, 0.4)

14. Know rules for parallel and perpendicular lines

1 Find the gradient of a straight line that is

a parallel to the line $y = 3 - 2x$, **b** parallel to the line $2x - 5y + 1 = 0$,

c perpendicular to the line $y = 3x + 4$, **d** perpendicular to the line $x + 2y - 3 = 0$.

2 Find, in the form $y = mx + c$, the equation of the straight line

a parallel to the line $y = 4x - 1$ which passes through the point with coordinates (1, 7),

b perpendicular to the line $y = 6 - x$ which passes through the point with coordinates (−4, 3),

c perpendicular to the line $x - 3y = 0$ which passes through the point with coordinates (−2, −2).

3 Find, in the form $ax + by + c = 0$, where a, b and c are integers, the equation of the straight line

a parallel to the line $2x - 3y + 5 = 0$ which passes through the point with coordinates (3, −1),

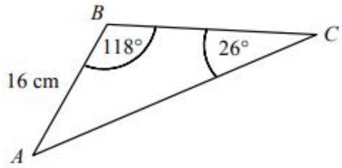
b perpendicular to the line $3x + 4y = 1$ which passes through the point with coordinates (2, 5),

c parallel to the line $3x + 5y = 6$ which passes through the point with coordinates (−4, −7).

Trigonometry

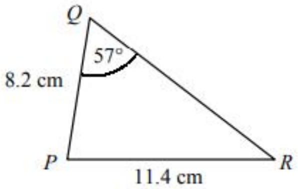
15. The sine rule

1



The diagram shows triangle ABC in which $AB = 16$ cm, $\angle ABC = 118°$ and $\angle ACB = 26°$.
Use the sine rule to find the length AC to 3 significant figures.

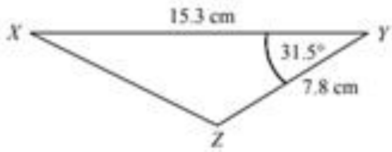
2



The diagram shows triangle PQR in which $PQ = 8.2$ cm, $PR = 11.4$ cm and $\angle PQR = 57°$.
Use the sine rule to find the size of $\angle PRQ$ in degrees to 1 decimal place.

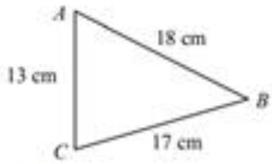
16. The cosine rule

1.



The diagram shows triangle XYZ in which $XY = 15.3$ cm, $YZ = 7.8$ cm and $\angle XYZ = 31.5°$.
Use the cosine rule to find the length XZ.

2.

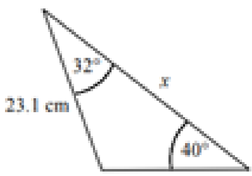


The diagram shows triangle ABC in which $AB = 18$ cm, $AC = 13$ cm and $BC = 17$ cm.
Use the cosine rule to find the size of $\angle ACB$.

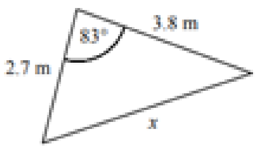
17. Mixed questions

1. Find the length x in each triangle.

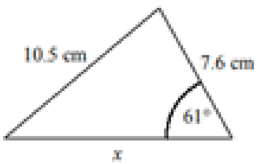
a



b

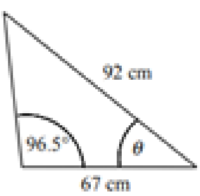


c

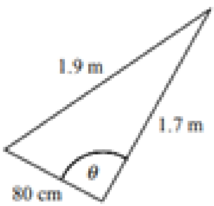


2. Find the angle θ in each triangle.

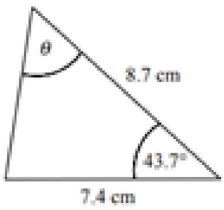
a



b



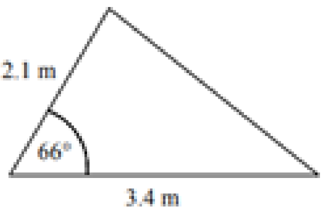
c



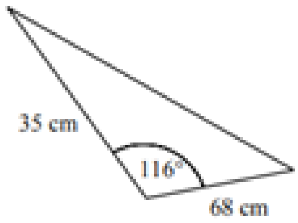
18. Area of triangles

1. Find the area of each of the following triangles.

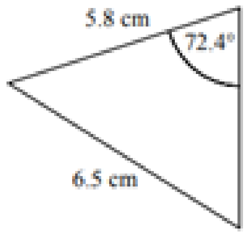
a



b



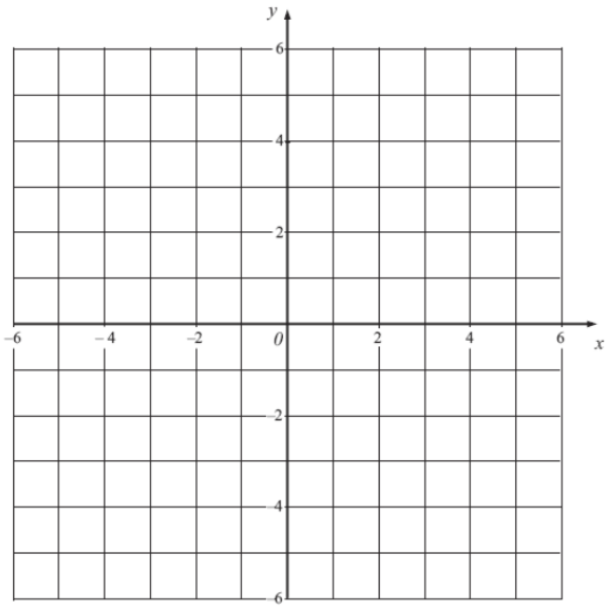
c



19. Inequalities on Graphs

1. On the grid, shade the region that satisfies all three of these inequalities

$y > -4$ $\quad$ $x < 2$ $\quad$ $y < 2x + 1$

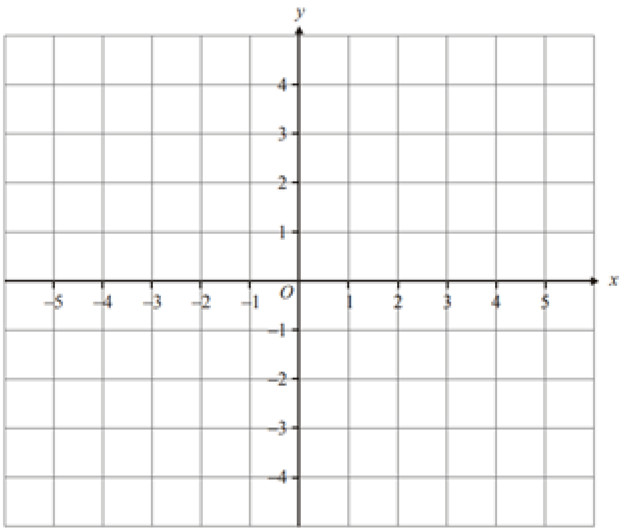


(Total for Question 19 = 4 marks)

2. $-2 < x \leq 1$ $\quad$ $y > -2$ $\quad$ $y < x + 1$

x and y are integers.

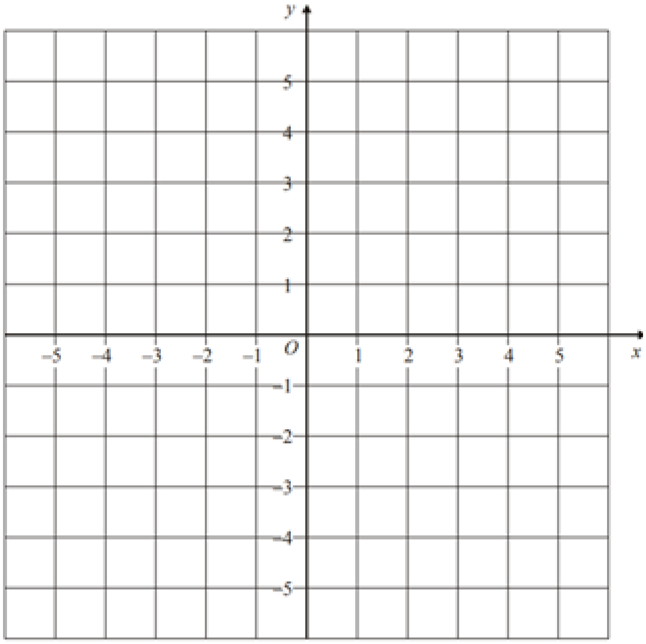
On the grid, mark with a cross (**×**), each of the six points which satisfies **all** these 3 inequalities.



3. $4x + 3y < 12$, $y < 3x$, $y > 0$, $x > 0$

x and y are both integers.

On the grid, mark with a cross (×), each of the **three** points which satisfy **all** these four inequalities.



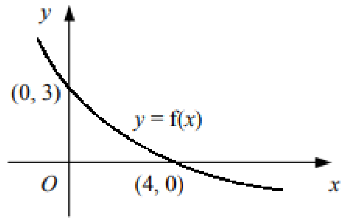
20. Transforming Graphs

1 Describe how the graph of $y = f(x)$ is transformed to give the graph of

a $y = f(x - 1)$ **b** $y = f(x) - 3$ **c** $y = 2f(x)$ **d** $y = f(4x)$

e $y = -f(x)$ **f** $y = \frac{1}{5}f(x)$ **g** $y = f(-x)$ **h** $y = f(\frac{2}{3}x)$

2



The diagram shows the curve with equation $y = f(x)$ which crosses the coordinate axes at the points (0, 3) and (4, 0).

Showing the coordinates of any points of intersection with the axes, sketch on separate diagrams the graphs of

a $y = 3f(x)$ **b** $y = f(x + 4)$ **c** $y = -f(x)$ **d** $y = f(\frac{1}{2}x)$

3 Find and simplify an equation of the graph obtained when

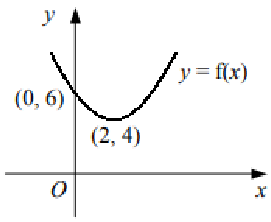
a the graph of $y = 2x + 5$ is translated by 1 unit in the positive y-direction,

b the graph of $y = 1 - 4x$ is stretched by a factor of 3 in the y-direction, about the x-axis,

c the graph of $y = 3x + 1$ is translated by 4 units in the negative x-direction,

d the graph of $y = 4x - 7$ is reflected in the x-axis.

4



The diagram shows the curve with equation $y = f(x)$ which has a turning point at (2, 4) and crosses the y-axis at the point (0, 6).

Showing the coordinates of the turning point and of any points of intersection with the axes, sketch on separate diagrams the graphs of

a $y = f(x) - 3$ **b** $y = f(x + 2)$ **c** $y = f(2x)$ **d** $y = \frac{1}{2}f(x)$

5 Describe a single transformation that would map the graph of $y = x^3$ onto the graph of

a $y = 4x^3$ **b** $y = (x - 2)^3$ **c** $y = -x^3$ **d** $y = x^3 + 5$

21. Algebraic Fractions

1 Simplify

a $\dfrac{3x-1}{18x-6}$ **b** $\dfrac{6x+15}{8x+20}$ **c** $\dfrac{3y+3}{y^2+7y+6}$ **d** $\dfrac{x^2-25}{x^2-7x+10}$

e $\dfrac{a^2-a-6}{a^2+3a-18}$ **f** $\dfrac{x^2+3x}{2x^2+5x-3}$ **g** $\dfrac{3t^2-11t-4}{t^2-16}$ **h** $\dfrac{6x^2-13x+6}{12x^2+x-6}$

2 Express as simply as possible

a $\dfrac{3x^2}{9x-9} \times \dfrac{4x-4}{2x}$ **b** $\dfrac{x^2-36}{x^2+7x+10} \div \dfrac{x-6}{x+2}$

c $\dfrac{n^2+2n}{n^2+6n+8} \times \dfrac{n+4}{n^2}$ **d** $\dfrac{4x-12}{x^2-4} \times \dfrac{x^2+2x}{x^2-2x-3}$

e $\dfrac{4y^2}{2y^2+y} \div \dfrac{y^2+2y-15}{2y^2+11y+5}$ **f** $\dfrac{x^2-1}{2x^2+7x-4} \times \dfrac{6x^2-5x+1}{3x^2-4x+1}$

g $\dfrac{10x-10}{5x+15} \div \dfrac{4-3x-x^2}{x^2+7x+12}$ **h** $\dfrac{a^3-3a^2}{8a^2-4a} \div \dfrac{a^2-9}{2a^2+5a-3}$

3 Express as a single fraction in its simplest form

a $\dfrac{2}{y} + \dfrac{7}{y+4}$ **b** $\dfrac{2x}{x-5} - \dfrac{1}{x+3}$ **c** $\dfrac{7}{x(x+2)} - \dfrac{3x}{x+2}$

d $\dfrac{x}{(x-3)(x-1)} + \dfrac{5}{2(x-1)}$ **e** $\dfrac{2}{q^2+3q} + \dfrac{5q}{4q+12}$ **f** $\dfrac{4}{3x-3} + \dfrac{x+2}{x^2-x}$

g $\dfrac{4}{x+5} + \dfrac{x}{x^2+8x+15}$ **h** $\dfrac{6x}{x^2-4} - \dfrac{3}{x+2}$ **i** $\dfrac{5t+12}{2t^2+7t+3} - \dfrac{4}{2t+1}$

4 Simplify

a $\dfrac{x^2-5x}{6x-30}$ **b** $\dfrac{16-x^2}{x^2+2x-8}$ **c** $\dfrac{2x^2-4x-6}{3x^2-12x+9}$ **d** $\dfrac{x^3-x}{2x^2-x-1}$

e $\dfrac{3x-x^2}{2x^2-18}$ **f** $\dfrac{x^3+x^2-2x}{3x^2+4x-4}$ **g** $\dfrac{2+5x-3x^2}{2x^2+x-10}$ **h** $\dfrac{x^4-5x^2+4}{x^2-x-2}$

5 Express as simply as possible

a $\dfrac{10x^2-10}{5x+10} \times \dfrac{x^2+6x+8}{x^2+5x+4}$ **b** $\dfrac{t^2-2t}{2t^2-t-6} \div \dfrac{9t^2-4}{6t^2+13t+6}$

c $\dfrac{2x^2+12x+10}{4x^2-7x+3} \div \dfrac{4x^2+20x}{4x^2-3x}$ **d** $\dfrac{8x^2+6x-9}{4x^2+12x+9} \times \dfrac{2x^2+3x}{6-8x}$

e $\dfrac{x^4+6x^2+5}{x^2-9} \times \dfrac{2x^2-6x}{4x^2+4}$ **f** $\dfrac{y^4-16}{5y^2+9y-2} \div \dfrac{y^2+4}{25y^2-10y+1}$

Answers: Algebra

1. <u>Expand single, double and triple brackets</u>

1. **a** LHS $= (3x+1)(x^2-2x+4)$
$$= 3x(x^2-2x+4)+(x^2-2x+4)$$
$$= 3x^3-6x^2+12x+x^2-2x+4$$
$$= 3x^3-5x^2+10x+4 = \text{RHS}$$

b LHS $= (1+2x-x^2)(1-2x+x^2)$
$$= (1-2x+x^2)+2x(1-2x+x^2)-x^2(1-2x+x^2)$$
$$= 1-2x+x^2+2x-4x^2+2x^3-x^2+2x^3-x^4$$
$$= 1-4x^2+4x^3-x^4 = \text{RHS}$$

c LHS $= (3-x)^3$
$$= (3-x)(9-6x+x^2)$$
$$= 3(9-6x+x^2)-x(9-6x+x^2)$$
$$= 27-18x+3x^2-9x+6x^2-x^3$$
$$= 27-27x+9x^2-x^3 = \text{RHS}$$

2. **a**
$$= x(x^2+5x-6)+(x^2+5x-6)$$
$$= x^3+5x^2-6x+x^2+5x-6$$
$$= x^3+6x^2-x-6$$

b
$$= 2x(x^2-3x+7)-5(x^2-3x+7)$$
$$= 2x^3-6x^2+14x-5x^2+15x-35$$
$$= 2x^3-11x^2+29x-35$$

c
$$= 4(2+5x-x^2)-7x(2+5x-x^2)$$
$$= 8+20x-4x^2-14x-35x^2+7x^3$$
$$= 7x^3-39x^2+6x+8$$

d
$$= (3x-2)(3x-2)^2 = (3x-2)(9x^2-12x+4)$$
$$= 3x(9x^2-12x+4)-2(9x^2-12x+4)$$
$$= 27x^3-36x^2+12x-18x^2+24x-8$$
$$= 27x^3-54x^2+36x-8$$

e
$$= x^2(2x^2-x+9)+3(2x^2-x+9)$$
$$= 2x^4-x^3+9x^2+6x^2-3x+27$$
$$= 2x^4-x^3+15x^2-3x+27$$

f
$$= 4x(x^4-3x^2+5x+2)-(x^4-3x^2+5x+2)$$
$$= 4x^5-12x^3+20x^2+8x-x^4+3x^2-5x-2$$
$$= 4x^5-x^4-12x^3+23x^2+3x-2$$

g
$$= x^2(x^2+3x+1)+2x(x^2+3x+1)+5(x^2+3x+1)$$
$$= x^4+3x^3+x^2+2x^3+6x^2+2x+5x^2+15x+5$$
$$= x^4+5x^3+12x^2+17x+5$$

h
$$= x^2(2x^2-x+4)+x(2x^2-x+4)-3(2x^2-x+4)$$
$$= 2x^4-x^3+4x^2+2x^3-x^2+4x-6x^2+3x-12$$
$$= 2x^4+x^3-3x^2+7x-12$$

i
$$= 3x^2(2x^2-4x-8)-5x(2x^2-4x-8)+2(2x^2-4x-8)$$
$$= 6x^4-12x^3-24x^2-10x^3+20x^2+40x+4x^2-8x-16$$
$$= 6x^4-22x^3+32x-16$$

j
$$= x^2(x^2+2x-6)+2x(x^2+2x-6)-6(x^2+2x-6)$$
$$= x^4+2x^3-6x^2+2x^3+4x^2-12x-6x^2-12x+36$$
$$= x^4+4x^3-8x^2-24x+36$$

k
$$= x^3(2x^4+x^2+3)+4x^2(2x^4+x^2+3)+(2x^4+x^2+3)$$
$$= 2x^7+x^5+3x^3+8x^6+4x^4+12x^2+2x^4+x^2+3$$
$$= 2x^7+8x^6+x^5+6x^4+3x^3+13x^2+3$$

l
$$= 6(3+x^2-x^3+2x^4)-2x(3+x^2-x^3+2x^4)+x^3(3+x^2-x^3+2x^4)$$
$$= 18+6x^2-6x^3+12x^4-6x-2x^3+2x^4-4x^5+3x^3+x^5-x^6+2x^7$$
$$= 2x^7-x^6-3x^5+14x^4-5x^3+6x^2-6x+18$$

3. **a**
$$= (p^2-1)(2p^2+11p+12)$$
$$= p^2(2p^2+11p+12)-(2p^2+11p+12)$$
$$= 2p^4+11p^3+12p^2-2p^2-11p-12$$
$$= 2p^4+11p^3+10p^2-11p-12$$

b
$$= t(t^2+3t+5)+2(t^2+3t+5)+t(t^2+t+7)+4(t^2+t+7)$$
$$= t^3+3t^2+5t+2t^2+6t+10+t^3+t^2+7t+4t^2+4t+28$$
$$= 2t^3+10t^2+22t+38$$

c
$$= 2x^2(x^2+x-4)-6(x^2+x-4)+3x(4x^3+2x^2-x+6)-(4x^3+2x^2-x+6)$$
$$= 2x^4+2x^3-8x^2-6x^2-6x+24+12x^4+6x^3-3x^2+18x-4x^3-2x^2+x-6$$
$$= 14x^4+4x^3-19x^2+13x+18$$

d
$$= u(u^3-4u^2-3)+2(u^3-4u^2-3)-2u^3(u^2+5u-3)-u(u^2+5u-3)+(u^2+5u-3)$$
$$= u^4-4u^3-3u+2u^3-8u^2-6-2u^5-10u^4+6u^3-u^3-5u^2+3u+u^2+5u-3$$
$$= -2u^5-9u^4+3u^3-12u^2+5u-9$$

2. Factorise linear, quadratic and simple cubic expressions

1. a $2(x+3)$ b $x(x+6)$ c $3y(2y-3)$ d $3xy^2z(8xy+5z^3)$

2. **a** $(x+1)(x+3)$ **b** $(x+2)(x+5)$ **c** $(y-1)(y-2)$ **d** $(x-3)^2$
e $(y+1)(y-2)$ **f** $(a+4)(a-2)$ **g** $(x+1)(x-1)$ **h** $(p+2)(p+7)$
i $(x+3)(x-5)$ **j** $(m-2)(m-8)$ **k** $(t+6)(t-3)$ **l** $(y-5)(y-8)$
m $(r+4)(r-4)$ **n** $(y+7)(y-9)$ **o** $(a+11)^2$ **p** $(x+12)(x-6)$
q $(x-2)(x-13)$ **r** $(s+8)(s+15)$ **s** $(p+17)(p-3)$ **t** $(m-10)(m+9)$

3. **a** $(2x+1)(x+1)$ **b** $(3p+1)(p+2)$ **c** $(2y-3)(y-1)$ **d** $(2+m)(1-m)$
e $(3r+1)(r-1)$ **f** $(5+y)(1-4y)$ **g** $(3a-1)(a-4)$ **h** $(5x+2)(x-2)$
i $(2x+1)(2x+3)$ **j** $(3s-1)^2$ **k** $(2m+5)(2m-5)$ **l** $(2+3y)(1-2y)$
m $(4u+1)(u+4)$ **n** $(3p+4)(2p-1)$ **o** $(8x+3)(x+2)$ **p** $(6r-5)(2r+3)$

4. a $x(x+3)(x+4)$ b $x(2x+1)(x-4)$ c $x(4x+1)(3x-2)$

3. Laws of indices

1. **a** $=\frac{1}{3^2}=\frac{1}{9}$ **b** $=1$ **c** $=\frac{1}{(-2)^6}=\frac{1}{64}$
d $=6^2=36$ **e** $=(\frac{3}{2})^{-3}=(\frac{2}{3})^3=\frac{8}{27}$ **f** $=\sqrt{9}=3$
g $=\sqrt[4]{16}=2$ **h** $=\sqrt[3]{-27}=-3$ **i** $=\sqrt{\frac{1}{49}}=\frac{1}{7}$
j $=\sqrt[3]{125}=5$ **k** $=\sqrt{\frac{4}{9}}=\frac{2}{3}$ **l** $=\frac{1}{\sqrt{36}}=\frac{1}{6}$
m $=\frac{1}{\sqrt[4]{81}}=\frac{1}{3}$ **n** $=\frac{1}{\sqrt[3]{-64}}=-\frac{1}{4}$ **o** $=\sqrt[5]{32}=2$
p $=\sqrt[3]{-\frac{8}{125}}=-\frac{2}{5}$ **q** $=\sqrt{\frac{9}{4}}=\frac{3}{2}$ or $1\frac{1}{2}$ **r** $=(\frac{27}{8})^{-\frac{1}{3}}=\sqrt[3]{\frac{8}{27}}=\frac{2}{3}$

2. **a** $=(\sqrt{4})^3=2^3=8$ **b** $=(\sqrt[3]{27})^2=3^2=9$
c $=(\sqrt[4]{16})^3=2^3=8$ **d** $=(\sqrt[3]{-125})^2=(-5)^2=25$
e $=(\sqrt{9})^5=3^5=243$ **f** $=\frac{1}{(\sqrt[3]{8})^2}=\frac{1}{2^2}=\frac{1}{4}$
g $=\frac{1}{(\sqrt{36})^3}=\frac{1}{6^3}=\frac{1}{216}$ **h** $=(\sqrt[3]{\frac{1}{8}})^4=(\frac{1}{2})^4=\frac{1}{16}$
i $=(\sqrt{\frac{4}{9}})^3=(\frac{2}{3})^3=\frac{8}{27}$ **j** $=(\sqrt[3]{216})^2=6^2=36$
k $=(\sqrt{\frac{16}{9}})^3=(\frac{4}{3})^3=\frac{64}{27}$ or $2\frac{10}{27}$ **l** $=(\sqrt[3]{-\frac{27}{64}})^4=(-\frac{3}{4})^4=\frac{81}{256}$
m $=\sqrt{\frac{4}{100}}=\frac{2}{10}=\frac{1}{5}$ or 0.2 **n** $=(\frac{9}{4})^{-\frac{3}{2}}=(\sqrt{\frac{4}{9}})^3=(\frac{2}{3})^3=\frac{8}{27}$
o $=(\sqrt[3]{\frac{64}{1000}})^2=(\frac{4}{10})^2=\frac{4}{25}$ or 0.16 **p** $=(\frac{25}{16})^{-\frac{3}{2}}=(\sqrt{\frac{16}{25}})^3=(\frac{4}{5})^3=\frac{64}{125}$
q $=(\frac{81}{16})^{\frac{3}{4}}=(\sqrt[4]{\frac{81}{16}})^3=(\frac{3}{2})^3=\frac{27}{8}$ or $3\frac{3}{8}$ **r** $=(\frac{64}{27})^{-\frac{4}{3}}=(\sqrt[3]{\frac{27}{64}})^4=(\frac{3}{4})^4=\frac{81}{256}$

3.
a $= \sqrt{4} \times \sqrt[3]{27}$ $= 2 \times 3 = 6$
b $= \sqrt[4]{16} + \sqrt{25}$ $= 2 + 5 = 7$
c $= \frac{1}{\sqrt[3]{8}} \div \sqrt{36}$ $= \frac{1}{2} \div 6 = \frac{1}{12}$
d $= \sqrt[3]{-64} \times (\sqrt{9})^3$ $= -4 \times 27 = -108$

e $= 3^2 - \sqrt[3]{-8}$ $= 9 - (-2) = 11$
f $= \sqrt{\frac{1}{25}} \times 4^2$ $= \frac{1}{5} \times 16 = \frac{16}{5}$ or $3\frac{1}{5}$
g $= (\sqrt[4]{81})^3 - \sqrt{49}$ $= 27 - 7 = 20$
h $= \sqrt[3]{27} \times (\sqrt{\frac{9}{4}})^3$ $= 3 \times \frac{27}{8} = \frac{81}{8}$ or $10\frac{1}{8}$

i $= \sqrt{9} \times (\sqrt[5]{-32})^3$ $= 3 \times (-8) = -24$
j $= \sqrt{121} + \sqrt[5]{32}$ $= 11 + 2 = 13$
k $= \sqrt{100} \div (\sqrt{\frac{1}{4}})^3$ $= 10 \div \frac{1}{8} = 80$
l $= \frac{1}{\sqrt[4]{16}} \times (\sqrt[5]{243})^2$ $= \frac{1}{2} \times 9 = \frac{9}{2}$ or $4\frac{1}{2}$

4.
a $= x^2$
b $= y^{-6}$
c $= 3p^{-4}$
d $= 8x^{-12}$
e $= y^{\frac{5}{2}}$
f $= 8b^{\frac{2}{3}+\frac{1}{4}} = 8b^{\frac{11}{12}}$
g $= x^{\frac{2}{3}-\frac{1}{5}} = x^{\frac{7}{15}}$
h $= a^{\frac{1}{2}-\frac{4}{3}} = a^{-\frac{5}{6}}$
i $= p^{\frac{1}{4}-(-\frac{1}{5})} = p^{\frac{9}{20}}$
j $= 9x^{\frac{4}{3}}$
k $= y^{1+\frac{2}{3}-\frac{4}{3}} = y^{\frac{1}{3}}$
l $= \frac{1}{3}t$
m $= b^{2+\frac{1}{4}-\frac{1}{2}} = b^{\frac{7}{4}}$
n $= y^{\frac{1}{2}+\frac{1}{3}-1} = y^{-\frac{1}{6}}$
o $= 2x^{\frac{2}{3}+(-\frac{1}{2})-\frac{1}{4}} = 2x^{-\frac{1}{12}}$
p $= \frac{1}{4}a^{1+\frac{1}{2}-(-\frac{1}{4})} = \frac{1}{4}a^{\frac{7}{4}}$

4. Simplify and use the rules of surds

1.
a $= \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$
b $= \sqrt{4} \times \sqrt{7} = 2\sqrt{7}$
c $= \sqrt{16} \times \sqrt{5} = 4\sqrt{5}$
d $= \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$
e $= \sqrt{4} \times \sqrt{6} = 2\sqrt{6}$
f $= \sqrt{64} \times \sqrt{2} = 8\sqrt{2}$
g $= \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$
h $= \sqrt{4} \times \sqrt{10} = 2\sqrt{10}$
i $= \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$
j $= \sqrt{16} \times \sqrt{7} = 4\sqrt{7}$
k $= \sqrt{9} \times \sqrt{11} = 3\sqrt{11}$
l $= \sqrt{49} \times \sqrt{3} = 7\sqrt{3}$
m $= \sqrt{36} \times \sqrt{6} = 6\sqrt{6}$
n $= \sqrt{400} \times \sqrt{2} = 20\sqrt{2}$
o $= \sqrt{36} \times \sqrt{5} = 6\sqrt{5}$
p $= \sqrt{4} \times \sqrt{15} = 2\sqrt{15}$
q $= \sqrt{121} \times \sqrt{3} = 11\sqrt{3}$
r $= \sqrt{16} \times \sqrt{13} = 4\sqrt{13}$

2.
a $= 3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$
b $= 4\sqrt{3} - 3\sqrt{3} = \sqrt{3}$
c $= 4\sqrt{2} + 6\sqrt{2} = 10\sqrt{2}$
d $= 6\sqrt{10} - 4\sqrt{10} = 2\sqrt{10}$
e $= 2\sqrt{5} - 3\sqrt{5} + 6\sqrt{5} = 5\sqrt{5}$
f $= 2\sqrt{6} + 5\sqrt{6} - 8\sqrt{6} = -\sqrt{6}$

3.
a $= 3 + 2\sqrt{3}$
b $= 4 - \sqrt{3} - 2 + 2\sqrt{3}$ $= 2 + \sqrt{3}$
c $= 2 + \sqrt{3} + 2\sqrt{3} + 3$ $= 5 + 3\sqrt{3}$
d $= 4 + 8\sqrt{3} + \sqrt{3} + 6$ $= 10 + 9\sqrt{3}$
e $= 27 - 24\sqrt{3} + 16$ $= 43 - 24\sqrt{3}$
f $= 6\sqrt{3} - 45 + 2 - 5\sqrt{3}$ $= -43 + \sqrt{3}$

4.
a $= 10 + 3\sqrt{5} + 2\sqrt{5} + 3$ $= 13 + 5\sqrt{5}$
b $= 4\sqrt{2} - 3 - 8 + 3\sqrt{2}$ $= 7\sqrt{2} - 11$
c $= 28 + 12\sqrt{7} + 9$ $= 37 + 12\sqrt{7}$
d $= 12 + 15\sqrt{2} - 2\sqrt{2} - 5$ $= 7 + 13\sqrt{2}$
e $= 5 + 2\sqrt{10} - \sqrt{10} - 4$ $= 1 + \sqrt{10}$
f $= (3 - 2\sqrt{2})(4 + \sqrt{2})$ $= 12 + 3\sqrt{2} - 8\sqrt{2} - 4$ $= 8 - 5\sqrt{2}$

5. Rationalise the denominator

1.
a $= \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{1}{5}\sqrt{5}$
b $= \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2}{3}\sqrt{3}$
c $= \frac{1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{1}{4}\sqrt{2}$
d $= \frac{14}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = 2\sqrt{7}$
e $= \frac{3\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \sqrt{6}$
f $= \frac{\sqrt{5}}{\sqrt{3}\sqrt{5}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{1}{3}\sqrt{3}$
g $= \frac{1}{3\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{1}{21}\sqrt{7}$
h $= \frac{12}{6\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$
i $= \frac{1}{4\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{1}{20}\sqrt{5}$
j $= \frac{3}{6\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{1}{12}\sqrt{6}$
k $= \frac{8\sqrt{5}}{9\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{4}{9}\sqrt{10}$
l $= \frac{15\sqrt{7}}{6\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{5}{6}\sqrt{21}$

2. a $= \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1$

b $= \frac{4}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{4(\sqrt{3}+1)}{3-1} = 2(\sqrt{3}+1)$

c $= \frac{1}{\sqrt{6}-2} \times \frac{\sqrt{6}+2}{\sqrt{6}+2} = \frac{\sqrt{6}+2}{6-4} = \frac{1}{2}(\sqrt{6}+2)$ or $\frac{1}{2}\sqrt{6}+1$

d $= \frac{3}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{3(2-\sqrt{3})}{4-3} = 3(2-\sqrt{3})$

e $= \frac{1}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}} = \frac{2-\sqrt{5}}{4-5} = \sqrt{5}-2$

f $= \frac{\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{\sqrt{2}(\sqrt{2}+1)}{2-1} = 2+\sqrt{2}$

g $= \frac{6}{\sqrt{7}+3} \times \frac{\sqrt{7}-3}{\sqrt{7}-3} = \frac{6(\sqrt{7}-3)}{7-9} = 3(3-\sqrt{7})$

h $= \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}} = \frac{3-2\sqrt{2}}{9-8} = 3-2\sqrt{2}$

i $= \frac{1}{4-2\sqrt{3}} \times \frac{4+2\sqrt{3}}{4+2\sqrt{3}} = \frac{4+2\sqrt{3}}{16-12} = \frac{1}{2}(2+\sqrt{3})$ or $1+\frac{1}{2}\sqrt{3}$

j $= \frac{3}{3\sqrt{2}+4} \times \frac{3\sqrt{2}-4}{3\sqrt{2}-4} = \frac{3(3\sqrt{2}-4)}{18-16} = \frac{3}{2}(3\sqrt{2}-4)$ or $\frac{9}{2}\sqrt{2}-6$

k $= \frac{2\sqrt{3}}{7-4\sqrt{3}} \times \frac{7+4\sqrt{3}}{7+4\sqrt{3}} = \frac{2\sqrt{3}(7+4\sqrt{3})}{49-48} = 2(7\sqrt{3}+12)$

l $= \frac{6}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{6(\sqrt{5}+\sqrt{3})}{5-3} = 3(\sqrt{5}+\sqrt{3})$

6. Solve quadratics using factorising, the quadratic formula and completing the square

Factorising

1. a $(x-1)(x-3)=0$, $x=1$ or 3

b $(x+4)(x+2)=0$, $x=-4$ or -2

c $(x+5)(x-1)=0$, $x=-5$ or 1

d $x^2-7x-8=0$, $(x+1)(x-8)=0$, $x=-1$ or 8

e $(x+5)(x-5)=0$, $x=-5$ or 5

f $x^2-x-42=0$, $(x+6)(x-7)=0$, $x=-6$ or 7

g $x^2-3x=0$, $x(x-3)=0$, $x=0$ or 3

h $(x+9)(x+3)=0$, $x=-9$ or -3

i $x^2+4x-60=0$, $(x+10)(x-6)=0$, $x=-10$ or 6

j $x^2-5x-14=0$, $(x+2)(x-7)=0$, $x=-2$ or 7

k $(2x-1)(x-1)=0$, $x=\frac{1}{2}$ or 1

l $x^2-x=6x-12$, $x^2-7x+12=0$, $(x-3)(x-4)=0$, $x=3$ or 4

m $3x^2+11x-4=0$, $(3x-1)(x+4)=0$, $x=-4$ or $\frac{1}{3}$

n $2x^2-3x-5=0$, $(2x-5)(x+1)=0$, $x=-1$ or $\frac{5}{2}$

o $4x^2-23x-6=0$, $(4x+1)(x-6)=0$, $x=-\frac{1}{4}$ or 6

p $6x^2-19x+10=0$, $(3x-2)(2x-5)=0$, $x=\frac{2}{3}$ or $\frac{5}{2}$

q $(2x+1)^2=0$, $x=-\frac{1}{2}$

r $3x^2-13x+12=0$, $(3x-4)(x-3)=0$, $x=\frac{4}{3}$ or 3

s $4x^2+20x+25=5-x$, $4x^2+21x+20=0$, $(4x+5)(x+4)=0$, $x=-4$ or $-\frac{5}{4}$

t $6x^2-21x=14x+6$, $6x^2-35x-6=0$, $(6x+1)(x-6)=0$, $x=-\frac{1}{6}$ or 6

Quadratic Formula

2

a $x = \frac{-4 \pm \sqrt{16-4}}{2}$
$x = \frac{-4 \pm 2\sqrt{3}}{2}$
$x = -2 \pm \sqrt{3}$

b $t = \frac{-8 \pm \sqrt{64+16}}{-2}$
$t = \frac{-8 \pm 4\sqrt{5}}{-2}$
$t = 4 \pm 2\sqrt{5}$

c $y = \frac{20 \pm \sqrt{400-364}}{2}$
$y = \frac{20 \pm 6}{2}$
$y = 7$ or 13

d $r = \frac{-2 \pm \sqrt{4+28}}{2}$
$r = \frac{-2 \pm 4\sqrt{2}}{2}$
$r = -1 \pm 2\sqrt{2}$

e $a = \frac{-18 \pm \sqrt{324-24}}{2}$
$a = \frac{-18 \pm 10\sqrt{3}}{2}$
$a = -9 \pm 5\sqrt{3}$

f $m^2 - 5m - 5 = 0$
$m = \frac{5 \pm \sqrt{25+20}}{2}$
$m = \frac{1}{2}(5 \pm 3\sqrt{5})$

g $x = \frac{-11 \pm \sqrt{121-108}}{2}$
$x = \frac{1}{2}(-11 \pm \sqrt{13})$

h $u = \frac{-6 \pm \sqrt{36-24}}{4}$
$u = \frac{-6 \pm 2\sqrt{3}}{4}$
$u = \frac{1}{2}(-3 \pm \sqrt{3})$

i $y = \frac{1 \pm \sqrt{1+20}}{-2}$
$y = -\frac{1}{2}(1 \pm \sqrt{21})$

j $2x^2 - 3x - 2 = 0$
$x = \frac{3 \pm \sqrt{9+16}}{4}$
$x = \frac{3 \pm 5}{4}$
$x = -\frac{1}{2}$ or 2

k $p = \frac{-7 \pm \sqrt{49-12}}{6}$
$p = \frac{1}{6}(-7 \pm \sqrt{37})$

l $t^2 - 14t - 14 = 0$
$t = \frac{14 \pm \sqrt{196+56}}{2}$
$t = \frac{14 \pm 6\sqrt{7}}{2}$
$t = 7 \pm 3\sqrt{7}$

m $r^2 + 14r - 9 = 0$
$r = \frac{-14 \pm \sqrt{196+36}}{2}$
$r = \frac{-14 \pm 2\sqrt{58}}{2}$
$r = -7 \pm \sqrt{58}$

n $6u^2 + 4u - 1 = 0$
$u = \frac{-4 \pm \sqrt{16+24}}{12}$
$u = \frac{-4 \pm 2\sqrt{10}}{12}$
$u = \frac{1}{6}(-2 \pm \sqrt{10})$

o $3y^2 - 18y - 4 = 0$
$y = \frac{18 \pm \sqrt{324+48}}{6}$
$y = \frac{18 \pm 2\sqrt{93}}{6}$
$y = 3 \pm \frac{1}{3}\sqrt{93}$

p $4x^2 - 8x - 11 = 0$
$x = \frac{8 \pm \sqrt{64+176}}{8}$
$x = \frac{8 \pm 4\sqrt{15}}{8}$
$x = 1 \pm \frac{1}{2}\sqrt{15}$

Completing the square

3

a $(y-2)^2 - 4 + 2 = 0$
$(y-2)^2 = 2$
$y - 2 = \pm\sqrt{2}$
$y = 2 \pm \sqrt{2}$

b $(p+1)^2 - 1 - 2 = 0$
$(p+1)^2 = 3$
$p + 1 = \pm\sqrt{3}$
$p = -1 \pm \sqrt{3}$

c $(x-3)^2 - 9 + 4 = 0$
$(x-3)^2 = 5$
$x - 3 = \pm\sqrt{5}$
$x = 3 \pm \sqrt{5}$

d $(r+5)^2 - 25 + 7 = 0$
$(r+5)^2 = 18$
$r + 5 = \pm\sqrt{18} = \pm 3\sqrt{2}$
$r = -5 \pm 3\sqrt{2}$

e $(x-1)^2 - 1 = 11$
$(x-1)^2 = 12$
$x - 1 = \pm\sqrt{12} = \pm 2\sqrt{3}$
$x = 1 \pm 2\sqrt{3}$

f $(a-6)^2 - 36 - 18 = 0$
$(a-6)^2 = 54$
$a - 6 = \pm\sqrt{54} = \pm 3\sqrt{6}$
$a = 6 \pm 3\sqrt{6}$

g $(m - \frac{3}{2})^2 - \frac{9}{4} + 1 = 0$
$(m - \frac{3}{2})^2 = \frac{5}{4}$
$m - \frac{3}{2} = \pm\frac{\sqrt{5}}{2}$
$m = \frac{1}{2}(3 \pm \sqrt{5})$

h $(t - \frac{7}{2})^2 - \frac{49}{4} + 9 = 0$
$(t - \frac{7}{2})^2 = \frac{13}{4}$
$t - \frac{7}{2} = \pm\frac{\sqrt{13}}{2}$
$t = \frac{1}{2}(7 \pm \sqrt{13})$

i $(u + \frac{7}{2})^2 - \frac{49}{4} = 44$
$(u + \frac{7}{2})^2 = \frac{225}{4}$
$u + \frac{7}{2} = \pm\frac{15}{2}$
$u = -\frac{7}{2} \pm \frac{15}{2}$
$u = -11$ or 4

j $y^2 - 2y + \frac{1}{2} = 0$
$(y-1)^2 - 1 + \frac{1}{2} = 0$
$(y-1)^2 = \frac{1}{2}$
$y - 1 = \pm\frac{1}{\sqrt{2}} = \pm\frac{1}{2}\sqrt{2}$
$y = 1 \pm \frac{1}{2}\sqrt{2}$

k $p^2 + 6p = -\frac{23}{3}$
$(p+3)^2 - 9 = -\frac{23}{3}$
$(p+3)^2 = \frac{4}{3}$
$p + 3 = \pm\frac{2}{\sqrt{3}} = \pm\frac{2}{3}\sqrt{3}$
$p = -3 \pm \frac{2}{3}\sqrt{3}$

l $x^2 + 6x = \frac{9}{2}$
$(x+3)^2 - 9 = \frac{9}{2}$
$(x+3)^2 = \frac{27}{2}$
$x + 3 = \pm\sqrt{\frac{27}{2}} = \pm\frac{3}{2}\sqrt{6}$
$x = -3 \pm \frac{3}{2}\sqrt{6}$

m $m^2 - m = 1$
$(m - \frac{1}{2})^2 - \frac{1}{4} = 1$
$(m - \frac{1}{2})^2 = \frac{5}{4}$
$m - \frac{1}{2} = \pm\frac{\sqrt{5}}{2}$
$m = \frac{1}{2}(1 \pm \sqrt{5})$

n $4x^2 - 28x + 49 = 0$
$x^2 - 7x + \frac{49}{4} = 0$
$(x - \frac{7}{2})^2 - \frac{49}{4} + \frac{49}{4} = 0$
$(x - \frac{7}{2})^2 = 0$
$x = \frac{7}{2}$

o $t^2 + \frac{1}{3}t = \frac{1}{3}$
$(t + \frac{1}{6})^2 - \frac{1}{36} = \frac{1}{3}$
$(t + \frac{1}{6})^2 = \frac{13}{36}$
$t + \frac{1}{6} = \pm\frac{\sqrt{13}}{6}$
$t = \frac{1}{6}(-1 \pm \sqrt{13})$

p $a^2 - \frac{7}{2}a + 2 = 0$
$(a - \frac{7}{4})^2 - \frac{49}{16} + 2 = 0$
$(a - \frac{7}{4})^2 = \frac{17}{16}$
$a - \frac{7}{4} = \frac{\sqrt{17}}{4}$
$a = \frac{1}{4}(7 \pm \sqrt{17})$

7. Solve linear simultaneous equations

1

a $3x = 2x + 1$
$x = 1$
$\therefore\ x = 1, y = 3$

b $x - 6 = \frac{1}{2}x - 4$
$x = 4$
$\therefore\ x = 4, y = -2$

c $2x + 6 = 3 - 4x$
$x = -\frac{1}{2}$
$\therefore\ x = -\frac{1}{2}, y = 5$

d subtracting
$y + 4 = 0$
$y = -4$
$\therefore\ x = 7, y = -4$

e $2x + 4y + 22 = 0$
$2x - 3y + 1 = 0$
subtracting
$7y + 21 = 0$
$y = -3$
$\therefore\ x = -5, y = -3$

f $6x + 6y + 8 = 0$
$15x - 6y - 15 = 0$
adding
$21x - 7 = 0$
$x = \frac{1}{3}$
$\therefore\ x = \frac{1}{3}, y = -\frac{5}{3}$

8. Solve quadratic simultaneous equations

1.

a subtracting
$x^2 - x - 2 = 0$
$(x + 1)(x - 2) = 0$
$x = -1$ or 2
$\therefore\ x = -1, y = 4$
or $x = 2, y = 7$

b adding
$2x^2 - 7x + 3 = 0$
$(2x - 1)(x - 3) = 0$
$x = \frac{1}{2}$ or 3
$\therefore\ x = \frac{1}{2}, y = -\frac{7}{2}$
or $x = 3, y = -6$

c $y = 2x - 5$
sub
$x^2 + (2x - 5)^2 = 25$
$x^2 - 4x = 0$
$x(x - 4) = 0$
$x = 0$ or 4
$\therefore\ x = 0, y = -5$
or $x = 4, y = 3$

d $y = 2x + 10$
sub.
$x^2 + 2x(2x + 10) + 15 = 0$
$x^2 + 4x + 3 = 0$
$(x + 3)(x + 1) = 0$
$x = -3$ or -1
$\therefore\ x = -3, y = 4$
or $x = -1, y = 8$

e $y = 1 - x$
sub.
$x^2 - 2x(1 - x) - (1 - x)^2 = 7$
$x^2 = 4$
$x = \pm 2$
$\therefore\ x = -2, y = 3$
or $x = 2, y = -1$

f $y = 1 - x$
sub.
$3x^2 - x - (1 - x)^2 = 0$
$2x^2 + x - 1 = 0$
$(2x - 1)(x + 1) = 0$
$x = -1$ or $\frac{1}{2}$
$\therefore\ x = -1, y = 2$
or $x = \frac{1}{2}, y = \frac{1}{2}$

g $y = 4 - x$
sub.
$2x^2 + x(4 - x) + (4 - x)^2 = 22$
$x^2 - 2x - 3 = 0$
$(x + 1)(x - 3) = 0$
$x = -1$ or 3
$\therefore\ x = -1, y = 5$
or $x = 3, y = 1$

h $x = 2y$
sub.
$(2y)^2 - 4y - y^2 = 0$
$3y^2 - 4y = 0$
$y(3y - 4) = 0$
$y = 0$ or $\frac{4}{3}$
$\therefore\ x = 0, y = 0$
or $x = \frac{8}{3}, y = \frac{4}{3}$

i $y = 3 - \frac{1}{2}x$
sub.
$x^2 + x(3 - \frac{1}{2}x) = 4$
$x^2 - 6x + 8 = 0$
$(x - 2)(x - 4) = 0$
$x = 2$ or 4
$\therefore\ x = 2, y = 0$
or $x = 4, y = -3$

j $y = 2x - 3$
sub.
$2x^2 + (2x - 3) - (2x - 3)^2 = 8$
$x^2 - 7x + 10 = 0$
$(x - 2)(x - 5) = 0$
$x = 2$ or 5
$\therefore\ x = 2, y = 1$
or $x = 5, y = 7$

k $y = 2x - 7$
sub.
$x^2 - x(2x - 7) + (2x - 7)^2 = 13$
$x^2 - 7x + 12 = 0$
$(x - 3)(x - 4) = 0$
$x = 3$ or 4
$\therefore\ x = 3, y = -1$
or $x = 4, y = 1$

l $y = 5 - 3x$
sub.
$x^2 - 5x + (5 - 3x)^2 = 0$
$2x^2 - 7x + 5 = 0$
$(2x - 5)(x - 1) = 0$
$x = 1$ or $\frac{5}{2}$
$\therefore\ x = 1, y = 2$
or $x = \frac{5}{2}, y = -\frac{5}{2}$

9. Solve linear inequalities

1

a $2x < 6$
$x < 3$

b $3x \geq 21$
$x \geq 7$

c $2x > 8$
$x > 4$

d $3x \leq 36$
$x \leq 12$

e $5x \geq -15$
$x \geq -3$

f $\frac{1}{3}x < 1$
$x < 3$

g $9x \geq 54$
$x \geq 6$

h $3x < -4$
$x < -\frac{4}{3}$

i $x < 14$

j $4x \leq -10$
$x \leq -\frac{5}{2}$

k $2 < 3x$
$x > \frac{2}{3}$

l $5 \geq \frac{1}{2}x$
$x \leq 10$

2

a $y > 7$

b $4p \leq 2$
$p \leq \frac{1}{2}$

c $6 < 2x$
$x > 3$

d $2a \geq 4$
$a \geq 2$

e $15 < 3u$
$u > 5$

f $2b \geq 9$
$b \geq \frac{9}{2}$

g $3x < -18$
$x < -6$

h $y \geq -13$

i $-20 \leq 4p$
$p \geq -5$

j $r - 2 > 6$
$r > 8$

k $3 - 6t \leq t - 4$
$7 \leq 7t$
$t \geq 1$

l $6 + 2x \geq 24 - 4x$
$6x \geq 18$
$x \geq 3$

m $7y + 21 - 6y + 2 < 0$
$y < -23$

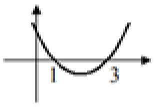
n $20 - 8x > 21 - 6x$
$-1 > 2x$
$x < -\frac{1}{2}$

o $12u - 3 - 5u + 15 < 9$
$7u < -3$
$u < -\frac{3}{7}$

10. Solve quadratic inequalities

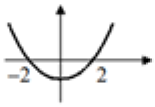
1.

a $(x-1)(x-3)<0$



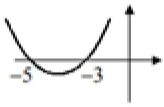
$\therefore 1<x<3$

b $(x+2)(x-2)\le 0$



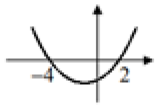
$\therefore -2\le x\le 2$

c $(x+5)(x+3)<0$



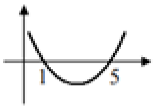
$\therefore -5<x<-3$

d $x^2+2x-8\le 0$
$(x+4)(x-2)\le 0$



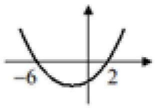
$\therefore -4\le x\le 2$

e $(x-1)(x-5)>0$



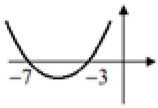
$\therefore x<1$ or $x>5$

f $x^2+4x-12>0$
$(x+6)(x-2)>0$



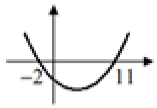
$\therefore x<-6$ or $x>2$

g $(x+7)(x+3)\ge 0$



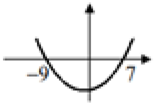
$\therefore x\le -7$ or $x\ge -3$

h $x^2-9x-22<0$
$(x+2)(x-11)<0$



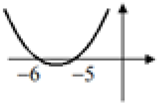
$\therefore -2<x<11$

i $x^2+2x-63\ge 0$
$(x+9)(x-7)\ge 0$



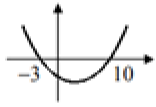
$\therefore x\le -9$ or $x\ge 7$

j $(x+6)(x+5)>0$



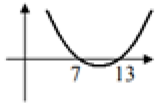
$\therefore x<-6$ or $x>-5$

k $x^2-7x-30<0$
$(x+3)(x-10)<0$



$\therefore -3<x<10$

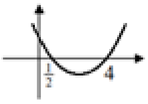
l $x^2-20x+91\ge 0$
$(x-7)(x-13)\ge 0$



$\therefore x\le 7$ or $x\ge 13$

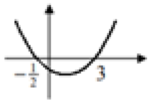
2.

a $(2x-1)(x-4)\le 0$



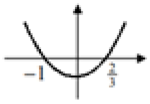
$\therefore \frac{1}{2}\le x\le 4$

b $(2r+1)(r-3)<0$



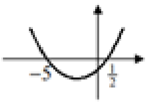
$\therefore -\frac{1}{2}<r<3$

c $3p^2+p-2\le 0$
$(3p-2)(p+1)\le 0$



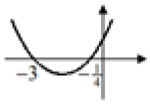
$\therefore -1\le p\le \frac{2}{3}$

d $(2y-1)(y+5)>0$



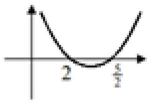
$\therefore y<-5$ or $y>\frac{1}{2}$

e $(4m+1)(m+3)<0$



$\therefore -3<m<-\frac{1}{4}$

f $2x^2-9x+10\ge 0$
$(2x-5)(x-2)\ge 0$



$\therefore x\le 2$ or $x\ge \frac{5}{2}$

Answers: Coordinated Geometry

11. Find the gradient of a line given two points

1

a $=\frac{5-1}{5-3}=2$ **b** $=\frac{9-7}{10-4}=\frac{1}{3}$ **c** $=\frac{5-1}{2-6}=-1$ **d** $=\frac{8-2}{2+2}=\frac{3}{2}$

e $=\frac{-1-3}{7-1}=-\frac{2}{3}$ **f** $=\frac{-7-5}{-5-4}=\frac{4}{3}$ **g** $=\frac{-8-0}{0+2}=-4$ **h** $=\frac{-2-6}{-7-8}=\frac{8}{15}$

12. Understand the equation of a straight line

1.

a grad $=4$
y-int $=-1$

b grad $=\frac{1}{3}$
y-int $=3$

c grad $=-1$
y-int $=6$

d grad $=-2$
y-int $=-\frac{3}{5}$

2.

a $y=-x-3$
grad $=-1$
y-int $=-3$

b $2y=x-6$
$y=\frac{1}{2}x-3$
grad $=\frac{1}{2}$
y-int $=-3$

c $3y=-3x+2$
$y=-x+\frac{2}{3}$
grad $=-1$
y-int $=\frac{2}{3}$

d $5y=4x+1$
$y=\frac{4}{5}x+\frac{1}{5}$
grad $=\frac{4}{5}$
y-int $=\frac{1}{5}$

13. Find the equation of a line

1.

a $y - 2 = 3(x - 1)$
$y = 3x - 1$

b $y - 3 = -(x - 5)$
$y = -x + 8$

c $y + 3 = 4(x + 2)$
$y = 4x + 5$

d $y - 1 = -2(x + 4)$
$y = -2x - 7$

e $y - 1 = \frac{1}{3}(x + 3)$
$y = \frac{1}{3}x + 2$

f $y + 2 = -\frac{5}{6}(x - 9)$
$y = -\frac{5}{6}x + \frac{11}{2}$

2.

a grad $= \frac{13-1}{4-0} = 3$
$y = 3x + 1$

b grad $= \frac{-1-9}{7-2} = -2$
$y - 9 = -2(x - 2)$
$y = -2x + 13$

c grad $= \frac{7-3}{2+4} = \frac{2}{3}$
$y - 3 = \frac{2}{3}(x + 4)$
$y = \frac{2}{3}x + \frac{17}{3}$

d grad $= \frac{8+2}{2+\frac{1}{2}} = 4$
$y - 8 = 4(x - 2)$
$y = 4x$

e grad $= \frac{-5+2}{18-3} = -\frac{1}{5}$
$y + 2 = -\frac{1}{5}(x - 3)$
$y = -\frac{1}{5}x - \frac{2}{5}$

f grad $= \frac{0.4-4}{-2+3.2} = -3$
$y - 4 = -3(x + 3.2)$
$y = -3x - 5.6$

14. Know rules for parallel and perpendicular lines

1

a grad of $y = 3 - 2x$ is -2
parallel grad $= -2$

b $2x - 5y + 1 = 0 \Rightarrow y = \frac{2}{5}x + \frac{1}{5}$
grad of $y = \frac{2}{5}x + \frac{1}{5}$ is $\frac{2}{5}$
parallel grad $= \frac{2}{5}$

c grad of $y = 3x + 4$ is 3
perp grad $= \frac{-1}{3} = -\frac{1}{3}$

d $x + 2y - 3 = 0 \Rightarrow y = \frac{3}{2} - \frac{1}{2}x$
grad of $y = \frac{3}{2} - \frac{1}{2}x$ is $-\frac{1}{2}$
perp grad $= \frac{-1}{-\frac{1}{2}} = 2$

2

a grad of $y = 4x - 1$ is 4
parallel grad $= 4$
$\therefore\ y - 7 = 4(x - 1)$
$y = 4x + 3$

b grad of $y = 6 - x$ is -1
perp grad $= 1$
$\therefore\ y - 3 = x + 4$
$y = x + 7$

c grad of $x - 3y = 0$ is $\frac{1}{3}$
perp grad $= -3$
$\therefore\ y + 2 = -3(x + 2)$
$y = -3x - 8$

3

a grad of $2x - 3y + 5 = 0$ is $\frac{2}{3}$
parallel grad $= \frac{2}{3}$
$\therefore\ y + 1 = \frac{2}{3}(x - 3)$
$3y + 3 = 2x - 6$
$2x - 3y - 9 = 0$

b grad of $3x + 4y = 1$ is $-\frac{3}{4}$
perp grad $= \frac{4}{3}$
$\therefore\ y - 5 = \frac{4}{3}(x - 2)$
$3y - 15 = 4x - 8$
$4x - 3y + 7 = 0$

c grad of $3x + 5y = 6$ is $-\frac{3}{5}$
parallel grad $= -\frac{3}{5}$
$\therefore\ y + 7 = -\frac{3}{5}(x + 4)$
$5y + 35 = -3x - 12$
$3x + 5y + 47 = 0$

Answers: Trigonometry

15. The sine rule

1 $\frac{AC}{\sin 118} = \frac{16}{\sin 26}$
$AC = \frac{16 \times \sin 118}{\sin 26}$
$= 32.2$ cm

2 $\frac{\sin \angle PRQ}{8.2} = \frac{\sin 57}{11.4}$
$\sin \angle PRQ = \frac{8.2 \times \sin 57}{11.4} = 0.6033$
$\angle PRQ = 37.1°$

16. The cosine rule

1. $XZ^2 = 7.8^2 + 15.3^2$
$- (2 \times 7.8 \times 15.3 \times \cos 31.5°)$
$= 91.422$
$XZ = 9.56$ cm (3sf)

2. $18^2 = 13^2 + 17^2 - (2 \times 13 \times 17 \times \cos \angle ACB)$
$\cos \angle ACB = \frac{13^2 + 17^2 - 18^2}{2 \times 13 \times 17}$
$= 0.3032$
$\angle ACB = 72.4°$ (1dp)

17. Mixed

1. **a** $\alpha = 180 - (40 + 32) = 108$

$\frac{x}{\sin 108} = \frac{23.1}{\sin 40}$

$x = \frac{23.1 \times \sin 108}{\sin 40}$

$x = 34.2$ cm (3sf)

b $x^2 = 2.7^2 + 3.8^2$

$- (2 \times 2.7 \times 3.8 \times \cos 83)$

$x^2 = 19.229$

$x = 4.39$ m (3sf)

c $\frac{\sin \alpha}{7.6} = \frac{\sin 61}{10.5}$

$\sin \alpha = \frac{7.6 \times \sin 61}{10.5} = 0.6331$

$\alpha = 39.276$

$\beta = 180 - (61 + 39.276) = 79.724$

$\frac{x}{\sin 79.724} = \frac{10.5}{\sin 61}$

$x = \frac{10.5 \times \sin 79.724}{\sin 61}$

$x = 11.8$ cm (3sf)

2. **a** $\frac{\sin \alpha}{67} = \frac{\sin 96.5}{92}$

$\sin \alpha = \frac{67 \times \sin 96.5}{92}$

$\sin \alpha = 0.7236$

$\alpha = 46.351$

$\theta = 180 - 96.5 - \alpha$

$\theta = 37.1°$ (1dp)

b $1.9^2 = 0.8^2 + 1.7^2$

$- (2 \times 0.8 \times 1.7 \times \cos \theta)$

$\cos \theta = \frac{0.8^2 + 1.7^2 - 1.9^2}{2 \times 0.8 \times 1.7}$

$\cos \theta = -0.02941$

$\theta = 91.7°$ (1dp)

c $l^2 = 7.4^2 + 8.7^2$

$- (2 \times 7.4 \times 8.7 \times \cos 43.7)$

$l^2 = 37.3608, l = 6.1123$

$\frac{\sin \theta}{7.4} = \frac{\sin 43.7}{6.1123}$

$\sin \theta = \frac{7.4 \times \sin 43.7}{6.1123} = 0.8364$

$\theta = 56.8°$ (1dp)

18. Area of triangles

1. **a** area

$= \frac{1}{2} \times 2.1 \times 3.4 \times \sin 66$

$= 3.26$ m^2 (3sf)

b area

$= \frac{1}{2} \times 35 \times 68 \times \sin 116$

$= 1070$ cm^2 (3sf)

c $\frac{\sin \alpha}{5.8} = \frac{\sin 72.4}{6.5}$

$\sin \alpha = \frac{5.8 \times \sin 72.4}{6.5} = 0.8505$

$\alpha = 58.270$

$\beta = 180 - (72.4 + \alpha) = 49.330$

area

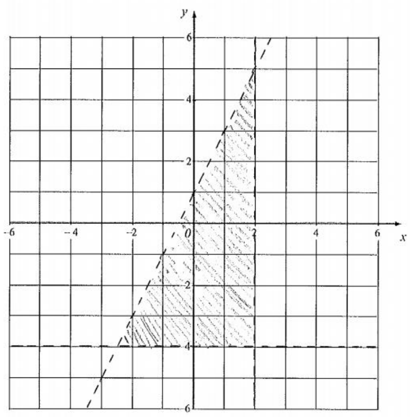
$= \frac{1}{2} \times 5.8 \times 6.5 \times \sin 49.330$

$= 14.3$ cm^2 (3sf)

Answers: Inequalities on graphs

1. On the grid, shade the region that satisfies all three of these inequalities

$y > -4$ $\quad x < 2$ $\quad y < 2x + 1$

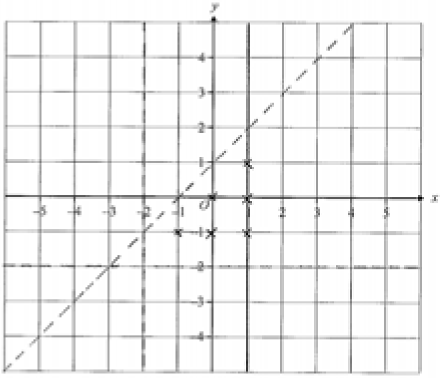


(Total for Question 19 = 4 marks)

2. $-2 < x \leq 1 \qquad y > -2 \qquad y < x + 1$

x and y are integers.

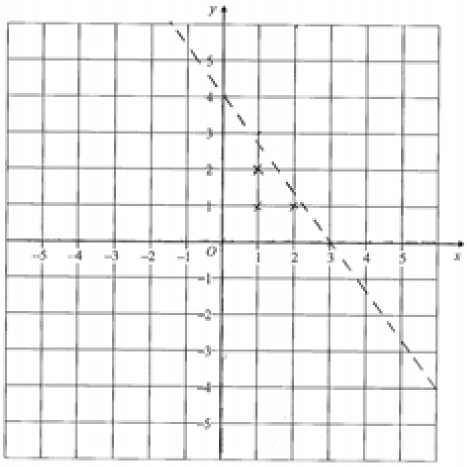
On the grid, mark with a cross (**×**), each of the six points which satisfies **all** these 3 inequalities.



3. $4x + 3y < 12, \qquad y < 3x, \qquad y > 0, \qquad x > 0$

x and y are both integers.

On the grid, mark with a cross (×), each of the **three** points which satisfy **all** these four inequalities.



Answers: Transforming graphs

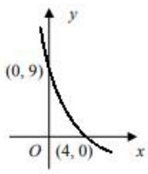
1 **a** translated 1 unit in positive x-direction **b** translated 3 units in negative y-direction

c stretched by a factor of 2 in y-direction **d** stretched by a factor of $\frac{1}{4}$ in x-direction

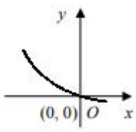
e reflected in the x-axis **f** stretched by a factor of $\frac{1}{5}$ in y-direction

g reflected in the y-axis **h** stretched by a factor of $\frac{3}{2}$ in x-direction

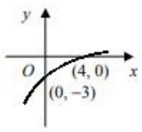
2 **a**



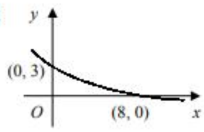
b



c



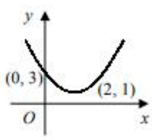
d



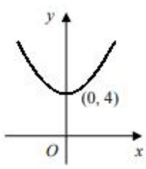
3 **a** $y = 2x + 5 + 1 \Rightarrow y = 2x + 6$ **b** $y = 3(1 - 4x) \Rightarrow y = 3 - 12x$

c $y = 3(x + 4) + 1 \Rightarrow y = 3x + 13$ **d** $y = -(4x - 7) \Rightarrow y = 7 - 4x$

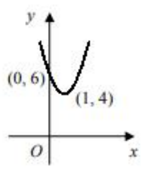
4 **a**



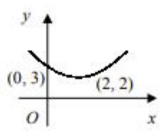
b



c



d



5 **a** stretch by a factor of 4 in y-direction **b** translation by 2 units in positive x-direction

c reflection in the x-axis **d** translation by 5 units in positive y-direction

Answers: Algebraic Fractions

1

a $= \dfrac{3x-1}{6(3x-1)} = \dfrac{1}{6}$

b $= \dfrac{3(2x+5)}{4(2x+5)} = \dfrac{3}{4}$

c $= \dfrac{3(y+1)}{(y+6)(y+1)} = \dfrac{3}{y+6}$

d $= \dfrac{(x+5)(x-5)}{(x-2)(x-5)} = \dfrac{x+5}{x-2}$

e $= \dfrac{(a+2)(a-3)}{(a+6)(a-3)} = \dfrac{a+2}{a+6}$

f $= \dfrac{x(x+3)}{(2x-1)(x+3)} = \dfrac{x}{2x-1}$

g $= \dfrac{(3t+1)(t-4)}{(t+4)(t-4)} = \dfrac{3t+1}{t+4}$

h $= \dfrac{(3x-2)(2x-3)}{(4x+3)(3x-2)} = \dfrac{2x-3}{4x+3}$

2

a $= \dfrac{3x^2}{9(x-1)} \times \dfrac{4(x-1)}{2x} = \dfrac{2x}{3}$

b $= \dfrac{(x+6)(x-6)}{(x+2)(x+5)} \times \dfrac{x+2}{x-6} = \dfrac{x+6}{x+5}$

c $= \dfrac{n(n+2)}{(n+4)(n+2)} \times \dfrac{n+4}{n^2} = \dfrac{1}{n}$

d $= \dfrac{4(x-3)}{(x+2)(x-2)} \times \dfrac{x(x+2)}{(x+1)(x-3)} = \dfrac{4x}{(x-2)(x+1)}$

e $= \dfrac{4y^2}{y(2y+1)} \times \dfrac{(2y+1)(y+5)}{(y+5)(y-3)} = \dfrac{4y}{y-3}$

f $= \dfrac{(x+1)(x-1)}{(2x-1)(x+4)} \times \dfrac{(3x-1)(2x-1)}{(3x-1)(x-1)} = \dfrac{x+1}{x+4}$

g $= \dfrac{10(x-1)}{5(x+3)} \times \dfrac{(x+3)(x+4)}{(4+x)(1-x)} = -2$

h $= \dfrac{a^2(a-3)}{4a(2a-1)} \times \dfrac{(2a-1)(a+3)}{(a+3)(a-3)} = \dfrac{a}{4}$

3

a $= \dfrac{2(y+4)+7y}{y(y+4)} = \dfrac{9y+8}{y(y+4)}$

b $= \dfrac{2x(x+3)-(x-5)}{(x-5)(x+3)} = \dfrac{2x^2+5x+5}{(x-5)(x+3)}$

c $= \dfrac{7-3x^2}{x(x+2)}$

d $= \dfrac{2x+5(x-3)}{2(x-3)(x-1)} = \dfrac{7x-15}{2(x-3)(x-1)}$

e $= \dfrac{2}{q(q+3)} + \dfrac{5q}{4(q+3)} = \dfrac{8+5q^2}{4q(q+3)}$

f $= \dfrac{4}{3(x-1)} + \dfrac{x+2}{x(x-1)} = \dfrac{4x+3(x+2)}{3x(x-1)} = \dfrac{7x+6}{3x(x-1)}$

g $= \dfrac{4}{x+5} + \dfrac{x}{(x+3)(x+5)} = \dfrac{4(x+3)+x}{(x+3)(x+5)} = \dfrac{5x+12}{(x+3)(x+5)}$

h $= \dfrac{6x}{(x+2)(x-2)} - \dfrac{3}{x+2} = \dfrac{6x-3(x-2)}{(x+2)(x-2)} = \dfrac{3x+6}{(x+2)(x-2)} = \dfrac{3(x+2)}{(x+2)(x-2)} = \dfrac{3}{x-2}$

i $= \dfrac{5t+12}{(2t+1)(t+3)} - \dfrac{4}{2t+1} = \dfrac{5t+12-4(t+3)}{(2t+1)(t+3)} = \dfrac{t}{(2t+1)(t+3)}$

4

a $= \dfrac{x(x-5)}{6(x-5)} = \dfrac{x}{6}$

b $= \dfrac{(4+x)(4-x)}{(x+4)(x-2)} = \dfrac{4-x}{x-2}$

c $= \dfrac{2(x-3)(x+1)}{3(x-1)(x-3)} = \dfrac{2(x+1)}{3(x-1)}$

d $= \dfrac{x(x+1)(x-1)}{(2x+1)(x-1)} = \dfrac{x(x+1)}{2x+1}$

e $= \dfrac{x(3-x)}{2(x+3)(x-3)} = -\dfrac{x}{2(x+3)}$

f $= \dfrac{x(x+2)(x-1)}{(3x-2)(x+2)} = \dfrac{x(x-1)}{3x-2}$

g $= \dfrac{(2-x)(1+3x)}{(2x+5)(x-2)} = -\dfrac{3x+1}{2x+5}$

h $= \dfrac{(x^2-1)(x^2-4)}{(x+1)(x-2)} = \dfrac{(x+1)(x-1)(x+2)(x-2)}{(x+1)(x-2)} = (x-1)(x+2)$

5

a $= \dfrac{10(x+1)(x-1)}{5(x+2)} \times \dfrac{(x+2)(x+4)}{(x+1)(x+4)} = 2(x-1)$

b $= \dfrac{t(t-2)}{(2t+3)(t-2)} \times \dfrac{(3t+2)(2t+3)}{(3t+2)(3t-2)} = \dfrac{t}{3t-2}$

c $= \dfrac{2(x+1)(x+5)}{(4x-3)(x-1)} \times \dfrac{x(4x-3)}{4x(x+5)} = \dfrac{x+1}{2(x-1)}$

d $= \dfrac{(4x-3)(2x+3)}{(2x+3)^2} \times \dfrac{x(2x+3)}{2(3-4x)} = -\dfrac{x}{2}$

e $= \dfrac{(x^2+1)(x^2+5)}{(x+3)(x-3)} \times \dfrac{2x(x-3)}{4(x^2+1)} = \dfrac{x(x^2+5)}{2(x+3)}$

f $= \dfrac{(y^2+4)(y+2)(y-2)}{(5y-1)(y+2)} \times \dfrac{(5y-1)^2}{y^2+4} = (y-2)(5y-1)$